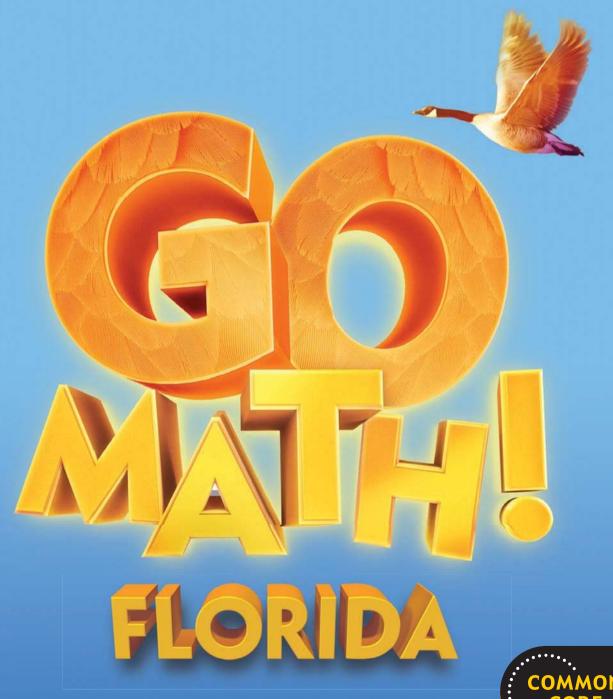
Reteach Book





Reteach Book

Grade 4



PROVIDES Tier 1 Intervention for Every Lesson



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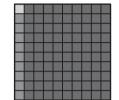
Getting Ready for Grade 5

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Model Place Value Relationships

A hundred grid can help you understand place-value relationships.

- One small square has been shaded to represent 1.
- Shade the rest of the first column. Count the number of small squares. There are <u>10</u> small squares. The model for 10 has <u>10</u> times as many squares as the model for <u>1</u>.



- Shade the remaining 9 columns. Count the number of small squares. There are <u>100</u> small squares. The model for 100 has <u>10</u> times as many squares as the model for <u>10</u>.
- If you shade ten hundred grids, you will have shaded 1,000 squares. So, the model for 1,000 has <u>10</u> times as many squares as the model for <u>100</u>.

A place-value chart helps you find the value of each digit in a number.

THOUSANDS			0	NES	
Hundreds	Tens	Ones	Hundreds Tens One		
		8	5	1	6

In the number 8.516:

The value of the digit 8 is 8 thousands, or 8,000.

The value of the digit 5 is 5 hundreds, or <u>500</u>.

The value of the digit 1 is 1 ten, or $\frac{10}{10}$.

The value of the digit 6 is 6 ones, or <u>6</u>.

Find the value of the underlined digit.

- **1.** <u>7</u>56
- **2.** 1,0<u>2</u>5
- **3.** <u>4</u>,279
- **4.** 35,703

Compare the values of the underlined digits.

5. <u>7</u>00 and <u>7</u>0

The value of 7 in _____ is ____

times the value of 7 in _____.

6. <u>5,000</u> and <u>5</u>00

The value of 5 in _____ is ____

times the value of 5 in _____.

Read and Write Numbers

Look at the digit 6 in the place-value chart below. It is in the hundred thousands place. So, its value is 6 hundred thousands.

In word form, the value of this digit is six hundred thousands.

In **standard form**, the value of the digit 6 is 600,000.



TH	OUSAND	S		ONES	
Hundreds	Tens	Ones	Hundreds	Tens	Ones
6	5	9,	0	5	8

Read the number shown in the place-value chart. In word form, this number is written as six hundred fifty-nine thousand, fifty-eight.

Note that when writing a number in words, a comma separates periods.

You can also write the number in **expanded form**: 600,000 + 50,000 + 9,000 + 50 + 8

Read and write each number in two other forms.

- **1.** 40,000 + 1,000 + 300 + 70 + 8
- 2. twenty-one thousand, four hundred
- **3.** 391,032

Compare and Order Numbers

Compare 31,072 and 34,318. Write <, >, or =.

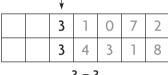
Step 1 Align the numbers by place value using grid paper.

Step 2 Compare the digits in each place value. Start at the greatest place.

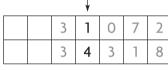
> Are the digits in the ten thousands place the same? Yes. Move to the thousands place.

Are the digits in the thousands place the same? No. 1 thousand is less than 4 thousands.

start here



3 = 3



1 < 4

Step 3 Use the symbols <, >, or = to compare the numbers.

< means is less than.

> means is greater than.

= means is equal to.

There are two ways to write the comparison.

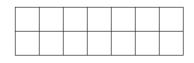
31,072 (<) 34,318

or 34,318 (>) 31,072

1. Use the grid paper to compare 21,409 and 20,891.

Write <.>. or =.

21.409



Compare. Write <, >, or =.

2. \$53.621

3. 82.550 (

Order from greatest to least.

4. 16,451; 16,250; 17,014

5. 561,028; 582,073; 549,006

Round Numbers

When you round a number, you replace it with a number that is easier to work with but not as exact. You can round numbers to different place values.

Round 478,456 to the place value of the underlined digit.

Step 1 Identify the underlined digit.

The underlined digit, 4, is in the hundred thousands place

Step 2 Look at the number to the right of the underlined digit.

If that number is 0-4, the underlined digit stays the same.

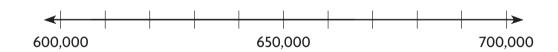
If that number is 5–9, the underlined digit is increased by 1.

The number to the right of the underlined digit is $\frac{7}{1}$, so the underlined digit, 4, will be increased by one; $4 + 1 = \frac{5}{1}$.

Step 3 Change all the digits to the right of the hundred thousands place to zeros.

So, 478,456 rounded to the nearest hundred thousand is 500,000.

1. In 2010, the population of North Dakota was 672,591 people. Use the number line to round this number to the nearest hundred thousand.



672,591 is closer to ______ than _____,

so it rounds to _____

Round to the place value of the underlined digit.

- **2.** 3,452
- **3.** <u>1</u>80

- **4.** \$<u>7</u>2,471
- **5.** <u>5</u>72,000

6. 950

- **7.** <u>6</u>,495
- **8.** 8<u>3</u>5,834
- **9.** 96,625

Rename Numbers

You can use place value to rename whole numbers. Here are different ways to name the number 1,400.

As thousands and hundreds

Think: $1.400 = \frac{1}{1}$ thousand $\frac{4}{1}$ hundreds. You can draw a quick picture to help.





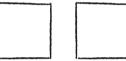


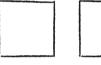




As hundreds

Think: $1.400 = \frac{14}{100}$ hundreds. You can draw a quick picture to help.



























As tens

Think: $1,400 = \frac{140}{100}$ tens.

As ones

Think: $1.400 = \frac{1,400}{1}$ ones.

Rename the number. Draw a quick picture to help.

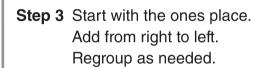
3.
$$6,000 =$$
 _____ thousands

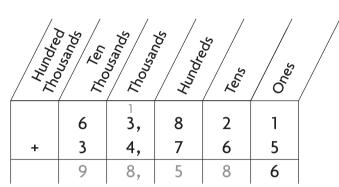
R5

Add Whole Numbers

Find the sum. 63,821 + 34,765

- Step 1 Round each addend to estimate. 60,000 + 30,000 = 90,000
- Step 2 Use a place-value chart to line up the digits by place value.

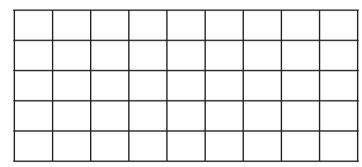




The sum is 98,586. Since 98.586 is close to the estimate 90,000, the answer is reasonable.

Estimate. Then find the sum.

1. Find 238,503 + 341,978. Use the grid to help.



Estimate: _____

- **2.** Estimate: _____
- **3.** Estimate: _____
- **4.** Estimate: _____

52,851 + 65,601

54,980 + 24,611

604,542 + 87,106

- **5.** Estimate: ______
- **6.** Estimate: ______ **7.** Estimate: _____

147,026 + 106,792

278,309 + 422,182

540,721 + 375,899

Subtract Whole Numbers

Find the difference. 5,128 - 3,956

Estimate first.

Think: 5,128 is close to 5,000. 3,956 is close to 4,000.

So, an estimate is 5,000 - 4,000 = 1,000.

Write the problem vertically. Use grid paper to align digits by place value.

Step 1 Subtract the ones.

	5,	ı	2	8	
_	3,	9	5	6	
				2	

$$8 - 6 = 2$$

Step 2 Subtract the tens.

		0	12		
	5,	X	2	8	
	3,	9	5	6	
			7	2	

There are not enough tens to subtract. Regroup 1 hundred as 10 tens. 12 tens - 5 tens = 7 tens

Step 3 Subtract the hundreds.

	4	10	12		
	5,	X	2	8	
	3,	9	5	6	
		I	7	2	

There are not enough hundreds to subtract. Regroup 1 thousand as 10 hundreds. 10 hundreds - 9 hundreds = 1 hundred

Step 4 Subtract the thousands.

	4	10	12		
	5,	X	2	8	
_	3,	9	5	6	
	I	ı	7	2	

4 thousands - 3 thousands =1 thousand

The difference is _____1,172 ___. Since 1,172 is close to the estimate of 1,000, the answer is reasonable.

Estimate. Then find the difference.

1. Estimate: ______ **2.** Estimate: _____

3. Estimate: _____

6.253 - 3.718

74.529 -38,453

232.318 -126,705

Problem Solving • Comparison Problemswith Addition and Subtraction

For a community recycling project, a school collects aluminum cans and plastic containers. This year the fourth grade collected 5,923 cans and 4,182 containers. This is 410 more cans and 24 more containers than the fourth grade collected last year. How many cans did the fourth grade collect last year?

Read the Problem						
What do I need to find?	What information do I need to use?	How will I use the information?				
I need to find the number of cans the fourth grade	The fourth grade students collected cans this					
collected last year.	They collected more cans this year than the fourth grade collected last year.	the fourth grade collected last year.				
Solve the Problem						
I can draw a bar model and write an equation to represent the problem.						
	5,923					
410						
	5,513					
5,923 - 410 = 5,513						
So, the fourth grade collected5,513 aluminum cans last year.						

Use the information above for 1 and 2.

- **1.** Altogether, how many aluminum cans and plastic containers did the fourth grade collect this year?
- 2. This year the fifth grade collected 216 fewer plastic containers than the fourth grade. How many plastic containers did the fifth grade collect?

Algebra • Multiplication Comparisons

Tara has 3 times as many soccer medals as Greg. Greg has 4 soccer medals. How many soccer medals does Tara have?

Step 1 Draw a model.



Step 2 Use the model to write an equation.

$$n = 3 \times 4$$
 Think: *n* is how many soccer medals Tara has.

Step 3 Solve the equation.

$$n = 12$$

So, Tara has 12 soccer medals.

Draw a model and write an equation.

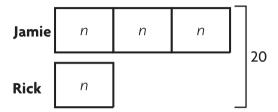
- **1.** 4 times as many as 7 is 28. **2.** 16 is 8 times as many as 2.

- **3.** 3 times as many as 6 is 18. **4.** 10 is 2 times as many as 5.

Algebra • Comparison Problems

Jamie has 3 times as many baseball cards as Rick. Together, they have 20 baseball cards. How many cards does Jamie have?

Step 1 Draw a box with the letter n in it to show that Rick has an unknown number of cards. Jamie has 3 times as many cards as Rick, so draw three identical boxes to represent Jamie's cards.



Step 2 Use the model to write an equation.

Think: There are 4 equal bars. The number in each bar is represented by n.

There are a total of 20 cards. So, $\underline{4} \times n = \underline{20}$.

Step 3 Solve the equation to find the value of n.

Think: 4 times what number is 20?

Since $4 \times \underline{5} = 20$, the value of *n* is $\underline{5}$.

Rick has ____5_ cards.

Step 4 Find how many cards Jamie has.

Think: Jamie has 3 times as many cards as Rick.

So, Jamie has $3 \times \underline{} = \underline{} = \underline{}$ baseball cards.

Draw a model. Write an equation and solve.

- 1. Maddie has 2 times as many stickers on her notebook as Meg. Together, they have 15 stickers. How many stickers are on Maddie's notebook?
- 2. How many more stickers are on Maddie's notebook than on Meg's notebook?

Multiply Tens, Hundreds, and Thousands

You can use a pattern to multiply with tens, hundreds, and thousands.

Count the number of zeros in the factors.

$$4 \times 6 = 24$$

← basic fact

$$4 \times 60 = 240$$

product is 0.

$$4 \times 600 = 2,400$$

 $4 \times 600 = 2,400 \leftarrow \text{When you multiply by hundreds, the last } \underline{\text{two}}$ digits in the product are 0.

$$4 \times 6,000 = 24,000$$

 $4 \times 6,000 = 24,000 \leftarrow$ When you multiply by thousands, the last <u>three</u> digits in the product are 0.

When the basic fact has a zero in the product, there will be an extra zero in the final product:

$$5 \times 4 = 20$$
, so $5 \times 4,000 = 20,000$

Complete the pattern.

1.
$$9 \times 2 = 18$$

3.
$$6 \times 6 = 36$$

2.
$$8 \times 4 = 32$$

4.
$$4 \times 7 = 28$$

Find the product.

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5.
$$7 \times 300 = 7 \times$$
 hundreds

5.
$$7 \times 300 = 7 \times$$
 hundreds **6.** $5 \times 8,000 = 5 \times$ thousands

Estimate Products

You can use rounding to estimate products.

Round the greater factor. Then use mental math to estimate the product.

6 × 95

Step 1 Round 95 to the nearest hundred.

95 rounds to 100.

Step 2 Use patterns and mental math.

$$6 \times 1 = 6$$

$$6 \times 10 = 60$$

$$6 \times 100 = 600$$

Find two numbers the exact answer is between.

7 × 759

Step 1 Estimate by rounding to the lesser hundred.

Think: $7 \times 7 = 49$ $7 \times 70 = 490$

 $7 \times 700 = 4,900$

$$7 \times 700 = 4,900$$

Step 2 Estimate by rounding to the greater hundred.

$$7 \times 759$$
 $7 \times 800 = 5,600$

Think: $7 \times 8 = 56$

$$7 \times 80 = 560$$

$$7 \times 800 = 5,600$$

So, the product is between 4,900 and 5,600.

Estimate the product by rounding.

2.
$$5 \times 29$$

3.
$$4 \times 703$$

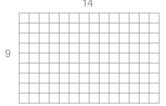
Estimate the product by finding two numbers the exact answer is between.

5.
$$7 \times 252$$

6.
$$8 \times 361$$

Multiply Using the Distributive Property

You can use rectangular models to multiply 2-digit numbers by 1-digit numbers.



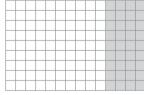
Find 9×14 .

Step 1 Draw a 9 by 14 rectangle on grid paper.

Step 2 Use the Distributive Property and products you know to break apart the model into two

smaller rectangles. **Think:** 14 = 10 + 4.

10



Step 3 Find the product each smaller rectangle represents.

$$9 \times 10 = 90$$

$$9 \times 4 = 36$$

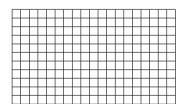
Step 4 Find the sum of the products.

$$90 + 36 = 126$$

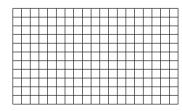
So, $9 \times 14 = 126$.

Model the product on the grid. Record the product.

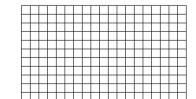
1. 3×13



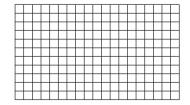
2. 6 × 16



3. 5×17



4. 4 × 14



Multiply Using Expanded Form

You can use expanded form or a model to find products.

Multiply. 3×26

Think and Write

Step 1 Write 26 in expanded form.

$$26 = 20 + 6$$

$$3 \times 26 = 3 \times (20 + 6)$$

Step 2 Use the Distributive Property.

$$3 \times 26 = (3 \times 20) + (\underline{} \times \underline{})$$

Step 3 Multiply the tens. Multiply the ones.

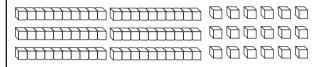
$$3 \times 26 = (3 \times 20) + (3 \times 6)$$

Step 4 Add the partial products. 78

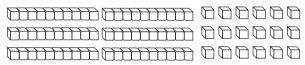
So,
$$3 \times 26 = \frac{78}{}$$

Use a Model

Step 1 Show 3 groups of 26.



Step 2 Break the model into tens and ones.



$$(3 \times 2 \text{ tens})$$
 $(3 \times 6 \text{ ones})$

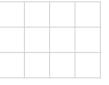
$$(3 \times 20)$$

$$(3 \times 6)$$

Step 3 Add to find the total product.

Record the product. Use expanded form to help.









Multiply Using Partial Products

Use partial products to multiply.

Multiply. $7 \times 332

Step 1 Estimate the product. 332 rounds to 300;
$$7 \times $300 = \frac{$2,100}{}$$
.

Step 3 Multiply the 3 tens, or 30, by 7.
$$\$332$$
 $\times 7$ or $\$210$

Step 5 Add the partial products.
$$$2,100 + $210 + $14 = \frac{$2,324}{}$$

So, $7 \times $332 = $2,324$. Since \$2,324 is close to the estimate of \$2,100, it is reasonable.

Estimate. Then record the product.

Multiply Using Mental Math

Use addition to break apart the larger factor.

Find 8×214 .

Think:
$$214 = 200 + 14$$

$$8 \times 214 = (8 \times 200) + (8 \times 14)$$

$$= 1,600 + 112$$

$$= 1,712$$

Use subtraction to break apart the larger factor.

Find 6×298 .

Think:
$$298 = 300 - 2$$

$$6 \times 298 = (6 \times 300) - (6 \times 2)$$

$$= 1,800 - 12$$

$$= 1,788$$

Use halving and doubling.

Find 14×50 .

Think: 14 can be evenly divided by 2.

$$14 \div 2 = _{-7}$$

$$7 \times 50 = 350$$

$$2 \times 350 = 700$$

When multiplying more than two numbers, use the Commutative Property to change the order of the factors.

Find $2 \times 9 \times 50$.

Think:
$$2 \times 50 = 100$$

$$2 \times 9 \times 50 = 2 \times \underline{50} \times 9$$
$$= \underline{100} \times 9$$
$$= \underline{900}$$

Find the product. Tell which strategy you used.

1.
$$5 \times 7 \times 20$$

2.
$$6 \times 321$$

3. 86×50

4. 9 × 399

Problem Solving • Multistep Multiplication Problems

Use the strategy *draw a diagram* to solve a multistep multiplication problem.

Amy planted 8 rows with 18 tulips in each row. In each of the 4 middle rows, there are 4 red tulips. All of the other tulips are yellow. How many of the tulips are yellow tulips?

Read the Problem	Solve the Problem		
What do I need to find?	I drew a diagram for each color of tulip.		
I need to find the total number of <u>yellow</u> tulips.	18 tulips RRRR		
What information do I need to use?	RRRR OTOWS		
There are8 rows of tulips with18 tulips in each row.	4 tulips		
There are 4 rows of tulips with 4	Next, I found the number in each section.		
red tulips in each row.	All Tulips Red Tulips		
How will I use the information?	$8 \times 18 = 144$ $4 \times 4 = 16$		
I can <u>multiply</u> to find the total number of tulips and the number of red tulips.	Last, I subtracted the number of red tulips from the total number of tulips.		
Then I can <u>subtract</u> to find the number	<u>144</u> _ <u>16</u> = <u>128</u>		
of yellow tulips.	So, there are 128 yellow tulips.		

- 1. A car dealer has 8 rows of cars with 16 cars in each row. In each of the first 3 rows, 6 are used cars. The rest of the cars are new cars. How many new cars does the dealer have?
- 2. An orchard has 4 rows of apple trees with 12 trees in each row. There are also 6 rows of pear trees with 15 trees in each row. How many apple and pear trees are in the orchard?

Multiply 2-Digit Numbers with Regrouping

Use place value to multiply with regrouping.

Multiply. 7×63

Step 1 Estimate the product.

$$7 \times 60 = 420$$

Step 2 Multiply the ones. Regroup 21 ones as 2 tens 1 one. Record the 1 one below the ones column and the 2 tens above the tens column.

 7×3 ones = 21 ones

Step 3 Multiply the tens. Then, add the regrouped tens. Record the tens.

44 tens = 4 hundreds 4 tens

 $7 \times 6 \text{ tens} = 42 \text{ tens}$

Add the 2 regrouped tens.

42 tens + 2 tens = 44 tens

So, $7 \times 63 = 441$. Since 441 is close to the estimate of 420, it is reasonable.

Estimate. Then record the product.

- 42
 × 6

 × 6
 37

 × 8
 × 8

 37
 × 8

 × 8
 × 9
- 37
 93
 86
 59

 × 5
 × 4
 × 9
 × 7

Multiply 3-Digit and 4-Digit **Numbers with Regrouping**

When you multiply 3-digit and 4-digit numbers, you may need to regroup.

Estimate. Then find the product.

Step 1 Estimate the product. \$1,324 rounds to $\$1,000; \$1,000 \times 7 = \$7,000$.

Step 2 Multiply the 4 ones by 7. Regroup the 28 ones as 2 tens 8 ones.

Step 3 Multiply the 2 tens by 7.

Add the regrouped tens.

$$\frac{\$1,324}{\times 7}$$

2 1 2

12

Step 4 Multiply the 3 hundreds by 7.

Regroup the 16 tens as 1 hundred 6 tens.

Add the regrouped hundred.

\$1,324 Regroup the 22 hundreds as 2 thousands

2 hundreds.

Step 5 Multiply the 1 thousand by 7. Add the regrouped thousands.

So. $7 \times \$1.324 = \9.268 .

Since \$9,268 is close to the estimate of \$7,000, the answer is reasonable.

Estimate. Then find the product.

Algebra • Solve Multistep Problems Using Equations

The **Order of Operations** is a special set of rules which gives the order in which calculations are done in an expression. First, multiply and divide from left to right. Then, add and subtract from left to right.

Use the order of operations to find the value of n.

$$6 \times 26 + 3 \times 45 - 11 = n$$

Step 1 Circle the first multiplication expression in the equation.

$$6 \times 26 + 3 \times 45 - 11 = n$$

Step 2 Multiply 6×26 .

$$156 + 3 \times 45 - 11 = n$$

Step 3 Circle the next multiplication expression in the equation.

$$156 + \cancel{3 \times 45} - 11 = n$$

Step 4 Multiply 3×45 .

$$156 + 135 - 11 = n$$

Step 5 There are no more multiplication or division expressions. Circle the first addition expression in the equation.

$$156 + 135 - 11 = n$$

Step 6 Add 156 + 135.

$$291 - 11 = n$$

Step 7 Subtract 291 - 11.

$$280 = n$$

Find the value of n.

1.
$$5 \times 43 + 9 \times 24 + 25 = n$$

2.
$$7 \times 29 + 4 \times 46 - 56 = n$$

$$\underline{\hspace{1cm}} = n$$

Multiply by Tens

One section of seating at an arena has 40 rows. Each row has 30 seats. How many seats in all are in that section?

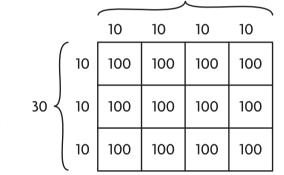
Multiply. 30×40

Step 1 Think of each factor as a multiple of 10 and as a repeated addition.

$$40 = 4 \times 10 \text{ or } 10 + 10 + 10 + 10$$

$$30 = 3 \times 10 \text{ or } 10 + 10 + 10$$

Step 2 Draw a diagram to show the multiplication.



40

Step 3 Each small square in the diagram shows 10×10 , or $\frac{100}{100}$. Count the squares.

There are 12 squares of 100.

Step 4 Use patterns and mental math to find 12×100 .

$$12 \times 1 = 12$$

$$12 \times 10 = 120$$

$$12 \times 100 = 1,200$$

There are $\frac{1,200}{1}$ seats in that section.

Choose a method. Then find the product.

1.
$$20 \times 90 =$$
 ______ **2.** $40 \times 40 =$ _____ **3.** $60 \times 70 =$ _____

4.
$$50 \times 30 =$$
 _____ **5.** $80 \times 60 =$ _____ **6.** $90 \times 40 =$ _____

Estimate Products

You can use rounding and compatible numbers to estimate products.

Use mental math and rounding to estimate the product.

Estimate. $62 \times 23

Step 1 Round each factor to the nearest ten. 62 rounds to 60.

\$23 rounds to **\$20**.

Step 2 Rewrite the problem using the rounded numbers. $60 \times \$20$

 $6 \times \$2 = \12 **Step 3** Use mental math.

 $6 \times \$20 = \120 $60 \times \$20 = \$1,200$

So, $62 \times 23 is about $\frac{$1,200}{}$

Use mental math and compatible numbers to estimate the product.

Estimate. 24×78

Step 1 Use compatible numbers. 25×80

Step 2 Use $25 \times 4 = 100$ to help find 25×8 . $25 \times 8 = 200$

Step 3 Since 80 has 1 zero, write 1 zero to the right of the product.

So, 24×78 is about ______.

 24×78 $25 \times 80 = 2.000$

Estimate the product. Choose a method.

1.
$$78 \times 21$$

1.
$$78 \times 21$$
 2. $59 \times 46 **3.** 81×33 **4.** 67×21

5.
$$88 \times $42$$
 6. 51×36 **7.** 73×73

6.
$$51 \times 36$$

7.
$$73 \times 73$$

9.
$$92 \times 19$$
 10. 26×37

11.
$$89 \times 18$$
 12. 58×59

Area Models and Partial Products

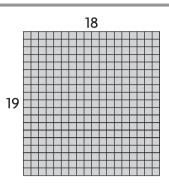
You can use area models to multiply 2-digit numbers by 2-digit numbers.

Draw a rectangle to find 19×18 .

The rectangle is 19 units long and 18 units wide.

Use the model and partial products to solve.

Step 1 Break apart the factors into tens and ones. Divide the area model into four smaller rectangles to show the factors.



10

Step 2 Find the products for each of the smaller rectangles.

$$10 \times 10 = 100$$
 $10 \times 8 = 80$ $9 \times 10 = 90$ $9 \times 8 = 72$

$$10 \times 8 = 80$$

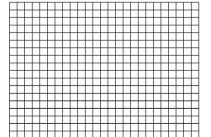
$$9 \times 10 = 90$$

$$9 \times 8 = 72$$

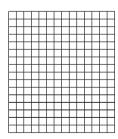
Step 3 Find the sum of the products. 100 + 80 + 90 + 72 = 342

So,
$$19 \times 18 = 342$$
.

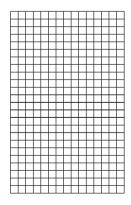
Draw a model to represent the product. Then record the product.



2.
$$16 \times 14$$



3.
$$24 \times 15$$



Multiply Using Partial Products

Multiply 25 \times 43. Record the product.

tens ones

3

Think: I can use partial products to find 25×43 .

Step 1 Multiply the tens by the tens.

 $20 \times 4 \text{ tens} = 80 \text{ tens}, \text{ or } 800.$

Step 2 Multiply the ones by the tens.

 20×3 ones = 60 ones, or 60.

Step 3 Multiply the tens by the ones.

 5×4 tens = 20 tens, or 200.

Step 4 Multiply the ones by the ones.

 5×3 ones = 15 ones, or 15.

Step 5 Add the partial products.

800 + 60 + 200 + 15 = 1,075.

So, $25 \times 43 = 1,075$

Record the product.

2.

59 × 38 3.

46 × 52 5.

6.

Multiply with Regrouping

Estimate. Then use regrouping to find 28×43 .

tens: 320 + 20 = 340.

Step 1 Round to estimate the product. $30 \times 40 = 1,200$

Step 2 Think: 28 = 2 tens 8 ones. 43 Multiply 43 by 8 ones. \times 28 $8 \times 3 = 24$. Record the 4. Write the 344 ← 8 × 43 regrouped 2 above the tens place. $8 \times 40 = 320$. Add the regrouped

2 43 Step 3 Multiply 43 by 2 tens. $20 \times 3 = 60$ and $20 \times 40 = 800$. \times 28 Record 860 below 344. 344 **860** ← 20 × 43 **1.204** ← 344 + 860

So. $28 \times 43 = 1,204$. 1,204 is close to 1,200. The answer is reasonable.

Estimate. Then find the product.

Step 4 Add the partial products.

1. Estimate: _____ **2.** Estimate: ______ **3.** Estimate: _____

36 43 51 \times 12 \times 29 \times 47

Lesson 3.6 Reteach

Choose a Multiplication Method

Estimate. Then use regrouping to find 47×89 .

Step 1 Estimate the product.
$$50 \times 90 = 4,500$$

$$50 \times 90 = 4,500$$

$$\begin{array}{c} 89 \\ \times 47 \\ \hline 3 \end{array}$$

3,560

So,
$$47 \times 89 = 4,183$$
. Since 4,183 is close to the estimate of 4,500, it is reasonable.

Estimate. Then choose a method to find the product.

Problem Solving • Multiply 2-Digit Numbers

A library ordered 17 cases with 24 books in each case. In 12 of the cases, 18 books were fiction books. The rest of the books were nonfiction. How many nonfiction books did the library order?

Read the Problem	Solve the Problem			
What do I need to find?	• First, find the total number of books ordered			
I need to find how many nonfiction	$17 \times 24 = 408$ books ordered			
books were ordered.	• Next, find the number of fiction books.			
What information do I need to use?	Last, draw a bar model. I need to subtract.			
17 cases of 24 books each were ordered.	408 books ordered			
In 12 cases, 18 books were fiction books.	216 fiction books			
How will I use the information?	?			
	408 - 216 = 192			
I can find the total number of books ordered and the number of fiction books ordered	So, the library ordered 192 nonfiction books.			
Then I can draw a bar model to compare the total number of books to the number of fiction books.				

- 1. A grocer ordered 32 cases with 28 small cans of fruit in each case. The grocer also ordered 24 cases with 18 large cans of fruit in each case. How many more small cans of fruit did the grocer order?
- 2. Rebecca rode her bike 16 miles each day for 30 days. Michael rode his bike 25 miles for 28 days. Who rode farther? How much farther?

Estimate Quotients Using Multiples

Find two numbers the quotient of 142 \div 5 is between. Then estimate the quotient.

You can use multiples to estimate. A **multiple** of a number is the product of a number and a counting number.

Step 1 Think: What number multiplied by 5 is about 142? Since 142 is greater than 10×5 , or 50, use counting numbers 10, 20, 30, and so on to find multiples of 5.

Step 2 Multiply 5 by multiples of 10 and make a table.

Counting Number	10	20	30	40
Multiple of 5	50	100	150	200

Step 3 Use the table to find multiples of 5 closest to 142.

$$20 \times 5 = \frac{100}{30 \times 5} = \frac{150}{150}$$
 142 is between $\frac{100}{100}$ and $\frac{150}{100}$.

142 is closest to $\frac{150}{1}$, so 142 ÷ 5 is about $\frac{30}{1}$.

Find two numbers the quotient is between. Then estimate the quotient.

- 1. $136 \div 6$
 - between _____ and ____

about _____

2. 95 ÷ 3

between _____ and ____

about _____

3. 124 ÷ 9

between _____ and ____

about _____

4. 238 ÷ 7

between ____ and ____

about _____

Remainders

Use counters to find the quotient and remainder.

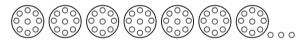
- Use 26 counters to represent the dividend, 26.
- Since you are dividing 26 by 9, draw 9 circles. Divide the 26 counters into 9 equal-sized groups.



• There are 2 counters in each circle, so the quotient is 2. There are 8 counters left over, so the remainder is 8.

Divide. Draw a quick picture to help.

- Use 66 counters to represent the dividend, 66.
- Since you are dividing 66 by 7, draw 7 circles.
 Divide 66 counters into 7 equal-sized groups.



• There are 9 counters in each circle, so the quotient is 9. There are 3 counters left over, so the remainder is 3.

Use counters to find the quotient and remainder.

1. 6)19

2. 3)14

Divide. Draw a quick picture to help.

3. 39 ÷ 4

4. 29 ÷ 3

Interpret the Remainder

When you solve a division problem with a remainder, the way you interpret the remainder depends on the situation and the question.

Way 1: Write the remainder as a fraction.

Callie has a board that is 60 inches long. She wants to cut 8 shelves of equal length from the board and use the entire board. How long will each shelf be?

Divide.
$$60 \div 8$$
 7 r4

The remainder, 4 inches, can be divided into 8 equal parts.

Write the remainder as a fraction.

Each shelf will be
$$\frac{7\frac{4}{8}}{8}$$
 inches long.

Way 2: Drop the remainder.

Callie has 60 beads. She wants to make 8 identical bracelets and use as many beads as possible on each bracelet. How many beads will be on each bracelet?

Divide.
$$60 \div 8$$
 7 r4

The remainder is the number of beads left over. Those beads will not be used. Drop the remainder.

Callie will use 7 beads on each bracelet.

Way 3: Add 1 to the quotient.

Callie has 60 beads. She wants to put 8 beads in each container. How many containers will she need?

Divide.
$$60 \div 8$$
 7 r4

The answer shows that Callie can fill 7 containers but will have 4 beads left over. She will need 1 more container for the 4 leftover beads. Add 1 to the quotient.

Callie will need 8 containers.

Way 4: Use only the remainder.

Callie has 60 stickers. She wants to give an equal number of stickers to 8 friends. She will give the leftover stickers to her sister. How many stickers will Callie give to her sister?

Divide.
$$60 \div 8$$
 $7 r4$

The remainder is the number of stickers left over. Use the remainder as the answer.

Callie will give her sister 4 stickers.

- Each van can hold 6 students. How many vans are needed?
- 1. There are 35 students going to the zoo. 2. Sue has 55 inches of ribbon. She wants to cut the ribbon into 6 equal pieces. How long will each piece be?

Divide Tens, Hundreds, and **Thousands**

You can use base-ten blocks, place value, and basic facts to divide.

Divide. 240 ÷ 3

Use base-ten b	Use place value.	
Step 1 Draw a quick picture to show 240.		Step 1 Identify the basic fact to use. Use $\underline{24 \div 3}$.
Step 2 You cannot divide 2 hundreds into 3 equal groups. Rename 2 hundreds as tens. 240 = tens		Step 2 Use place value to rewrite 240 as tens. $240 = \underline{24} \text{tens}$
Step 3 Separate the tens into 3 equal groups to divide. There are 3 groups of8 Write the answer. 240 ÷ 3 =80	tens.	Step 3 Divide. $24 \text{ tens} \div 3 = \underline{8} \text{ tens}$ $= \underline{80}$ Write the answer. $240 \div 3 = \underline{80}$

Use basic facts and place value to find the quotient.

What division fact can you use?

What division fact can you use?

28 tens \div 4 = _____ tens

18 hundreds \div 9 = ____ hundreds

Estimate Quotients Using Compatible Numbers

Compatible numbers are numbers that are easy to compute mentally. In division, one compatible number divides evenly into the other. Think of the multiples of a number to help you find compatible numbers.

Estimate. 6 216

Step 1 Think of these multiples of 6:

12 6

18

24

30

36

42

48

54

Find multiples that are close to the first 2 digits of the dividend. 18 tens and 24 tens are both close to 21 tens. You can use either or both numbers to estimate the quotient.

Step 2 Estimate using compatible numbers.

$$\frac{1}{180 \div 6} = 30$$

$$\frac{1}{240 \div 6} = 40$$

So. 216 \div 6 is between 30 and 40.

Step 3 Decide whether the estimate is closer to 30 or 40.

$$216 - 180 = 36$$

$$216 - 180 = 36$$
 $240 - 216 = 24$

216 is closer to 240, so use $\frac{40}{2}$ as the estimate.

Use compatible numbers to estimate the quotient.

1. 3)252

2. 6)546

3. 4)2,545

4. 5)314

5. 2)1,578

6. 8)289

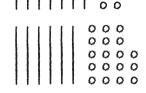
Division and the Distributive Property

Divide. $78 \div 6$

Use the Distributive Property and quick pictures to break apart numbers to make them easier to divide.

Step 1 Draw a guick picture to show 78.

Step 2 Think about how to break apart 78. You know 6 tens \div 6 = 10, so use 78 = 60 + 18. Draw a quick picture to show 6 tens and 18 ones.



Step 3 Draw circles to show 6 tens ÷ 6 and 18 ones ÷ 6. Your drawing shows the use of the Distributive Property. $78 \div 6 = (60 \div 6) + (18 \div 6)$



Step 4 Add the quotients to find $78 \div 6$.

$$78 \div 6 = (60 \div 6) + (18 \div 6)$$

$$= 10 + 3$$

$$= 13$$

Use quick pictures to model the quotient.

3.
$$68 \div 2 =$$

4.
$$65 \div 5 =$$

4.
$$65 \div 5 =$$
 5. $96 \div 8 =$ **6.** $90 \div 6 =$ **...**

Divide Using Repeated Subtraction

You can use repeated subtraction to divide. Use repeated subtraction to solve the problem.

Nestor has 27 shells to make bracelets. He needs 4 shells for each bracelet. How many bracelets can he make?

Divide. 27 ÷ 4

Write 4)27.

Step 1
Subtract the divisor until the remainder is less than the divisor.
Record a 1 each time you subtract.

Step 2

Count the number of times you subtracted the divisor, 4.

4 is subtracted six times with 3 left.

So, Nestor can make 6 bracelets. He will have 3 shells left.

Use repeated subtraction to divide.

Divide Using Partial Quotients

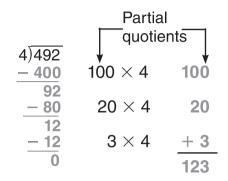
You can use partial quotients to divide.

Divide. 492 ÷ 4

Step 1 Subtract greater multiples of the divisor. Repeat if needed.

Step 2 Subtract lesser multiples of the divisor. Repeat until the remaining number is less than the divisor.

Step 3 Add the partial quotients.



Use rectangular models to record partial quotients.

	100	492
4	400	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$
	100	20 92
4	400	$\begin{vmatrix} 80 & 12 \end{vmatrix} = \frac{32}{12}$
	100	20 3
4	400	80 12

Divide. Use partial quotients.

Divide. Use rectangular models to record the partial quotients.

2. 852 ÷ 6 = ____

Model Division with Regrouping

You can use base-ten blocks to model division with regrouping.

Use base-ten blocks to find the quotient $65 \div 4$.

Step 1 Show 65 with base-ten blocks.

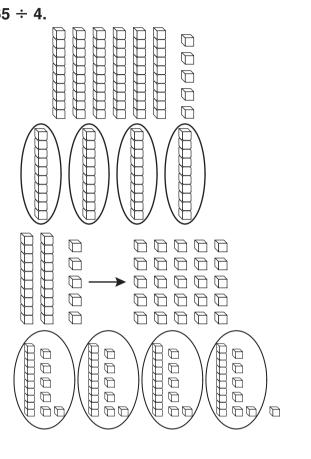
Step 2 Draw 4 circles to represent dividing 65 into 4 equal groups. Share the tens equally among the 4 groups.

Step 3 Regroup leftover tens as ones.

Step 4 Share the ones equally among the 4 groups.

There are $\underline{1}$ ten(s) and $\underline{6}$ one(s) in each group with $\underline{1}$ left over.

So, the quotient is 16 r1.



Divide. Use base-ten blocks.

2. 74 ÷ 3

Reteach

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Divide 7 hundreds by 3.

– Multiply. 3 imes 2 hundreds

Place the First Digit

Divide. 763 ÷ 3 = ■

Step 1 Estimate. Then divide the hundreds.

Think: 3×1 hundred = 3 hundreds 3×2 hundreds = 6 hundreds 3×3 hundreds = 9 hundreds

3 × 3 hundreds is too large.
Use 2 hundreds as an estimate.

Step 2 Bring down the tens digit. Then divide the tens.

 $\begin{array}{c}
2\\
3)763\\
\underline{-6} \downarrow\\
16
\end{array}$ Bring down the 6.

25 Divide 16 tens by 3.

- Subtract.

3)763

Step 3 Bring down the ones digit. Then divide the ones.

25 3)763 - 6 16 - 15 ▼ 13 ← Bring down the 3. 254 ← Divide 13 ones by 3.
3)763
- 6
16
- 15
13

12 ← Multiply. 3 × 4 ones

1 ←— Subtract.

Step 4 Check to make sure that the remainder is less than the divisor. Write the answer.

254 r1 1 < 3

Divide.

- **1.** 2)531
- **2.** 4)628
- **3.** 9)349
- **4.** 7)794

Divide by 1-Digit Numbers

Divide. $766 \div 6 = \blacksquare$

Step 1 Use place value to place the first digit.

Think: 7 hundreds can be shared among

6 groups without regrouping.

— Divide 16 tens by 6. Step 2 Bring down the 6)766 tens digit. Then divide Bring down the tens. - Multiply. 6 imes 2 tens

- Subtract. Divide 46 ones by 6.

the 6.

Step 3 Bring down the ones digit. Then divide the ones.

6)766 6)766 - 6 16 Bring down — 42 ← Multiply. 6 × 7 ones the 6. ← Subtract.

Step 4 Check to make sure that the remainder is less than the divisor. Write the answer.

4 < 6

Step 5 Use multiplication and addition to check your answer.

Divide and check.

1. 4)868

2. 2)657

3. 7) 8,473

Grade 4

Problem Solving • Multistep Division Problems

There are 72 third graders and 84 fourth graders going on a field trip. An equal number of students will ride on each of 4 buses. How many students will ride on each bus?

Read the Problem	Solve the Problem		
What do I need to find? I need to find the number of students who	I can model the number of students in all using a bar diagram.		
will ride on each bus. What information do I need to use?	72 84		
There are third graders and fourth graders. There will be buses.	I can model the number of buses and divide to find the number of students on		
How will I use the information?	each bus.		
I will make a bar diagram for each step. I will add 72 and 84 to find the total number	39 39 39 39		
of students. I will divide by4 to find how many students will ride on each bus.	156		
	So, <u>39</u> students will ride on each bus.		

Miranda has 180 beads for making jewelry. She buys 240 more beads. She wants to store the beads in a case with 6 sections. She wants to put the same number of beads in each section. How many beads should Miranda put in each section?

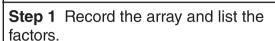
2. All 203 students at Polk School eat lunch at the same time. One day 19 students were absent. If 8 students sit at each table in the lunchroom, how many tables were used that day at lunch?

Lesson 5.1

Reteach

Model Factors

Use tiles to find all the factors of 25. Record the arrays and write the factors shown.



 $1 \times 25 = 25$

Think: Every whole number greater than 1

Factors: 1, 25

has at least two factors, that number and 1. Step 2 Make an array to see if 2 is a factor

of 25.

Think: An array has the same number of tiles in every row and the same number of

You cannot use all 25 tiles to make an array that has 2 rows. There is 1 tile left.

So, 2 is not a factor of 25.

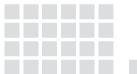
Step 3 Continue making arrays, counting by 1, to find all the other factors of 25.

Is 3 a factor?

tiles in every column.



Is 4 a factor?



3 rows, 1 tile left

No, 3 is not a factor of 25.

4 rows, 1 tile left

No. 4 is not a factor of 25.

Is 5 a factor?



5 rows, all tiles used.

 $5 \times 5 = 25$

There are the same number of tiles in each row and column. Yes, 5 is a factor of 25.

If you continue to make arrays up to 24,

you will find there are no additional factors of 25.

So, the factors of 25 are 1, 5, and 25.

Two factors that make a product are sometimes called a factor pair.

What are the factor pairs for 25? 1 and 25, 5 and 5

Use tiles to find all the factors of the product. Record the arrays and write the factors shown.

1. 35

2. 36

Factors and Divisibility

A number is divisible by another number if the quotient is a counting number and the remainder is 0. You can decide if a number is divisible by 2, 3, 5, 6, or 9 by using divisibility rules instead of dividing. Divisibility rules help you decide if one number is a factor of another.

Is 39 divisible by 2, 3, 5, 6, or 9?

Divisibility Rules

$$39 \div 2 = 19 \text{ r1} \rightarrow 39 \text{ is not divisible by } \underline{2}.$$

The last digit, 9, is not even, so 39 is not divisible by 2.

$$39 \div 3 = 13 \text{ r0} \rightarrow 39 \text{ is divisible by } 3$$
.

The sum of the digits, 3 + 9 = 12, is divisible by 3, so 39 is divisible by 3.

$$39 \div 5 = 7 \text{ r4} \rightarrow 39 \text{ is not divisible by } \underline{5}.$$

The last digit, 9, is not a 0 or 5, so 39 is not divisible by 5.

$$39 \div 6 = 6 \text{ r3} \rightarrow 39 \text{ is not divisible by } \underline{6}.$$

39 is not divisible by both 2 and 3, so it is not divisible by 6.

$$39 \div 9 = 4 \text{ r3} \rightarrow 39 \text{ is not divisible by } \underline{9}.$$

The sum of the digits, 3 + 9 = 12, is not divisible by 9, so 39 is not divisible by 9.

39 is divisible by 3. 3 is a factor of 39.

Tell whether 30 is divisible by 2, 3, 5, 6, or 9. Show your work.

3. 30 ÷ 5

Is 4 a factor of the number? Write yes or no.

Problem Solving • Common Factors

Susan sorts a collection of beads. There are 35 blue, 49 red, and 21 pink beads. She arranges all the beads into rows. Each row will have the same number of beads, and all the beads in a row will be the same color. How many beads can she put in each row?

Read the Problem	Solve the Problem
What do I need to find?	
of beads in each row,	Factors Factors of 35 of 49 of 21
if each row is equal	<u>1</u> <u>1</u> <u>1</u>
	<u>5</u> <u>7</u> <u>3</u>
and has only one	<u>7</u> <u>49</u> <u>7</u>
color	<u>35</u> <u>21</u>
What information do I need to use? Susan has and 21 pink beads	The common factors are and
How will I use the information?	
I can make a list to find all of the factors of 35, 49, and 21	So, Susan can put or
Then I can use the list to find the common factors	beads in each row.

- 1. Allyson has 60 purple buttons, 36 black buttons, and 24 green buttons. She wants to put all of the buttons in bins. She wants each bin to have only one color and all bins to have the same number of buttons. How many buttons can Allyson put in one bin?
- 2. Ricardo has a marble collection with 54 blue marbles, 24 red marbles, and 18 yellow marbles. He arranges the marbles into equal rows. The marbles in each row will be the same color. How many marbles can he put in one row?

Factors and Multiples

You know that $1 \times 10 = \underline{10}$ and $2 \times 5 = \underline{10}$.

So, 1, 2, 5, and 10 are all factors of 10.

You can skip count to find multiples of a number:

Count by 1s: 1, 2, 3, 4, 5, 6, 7, 8, 9, **10,** . . .

Count by 2s: 2, 4, 6, 8, **10,** 12, . . .

Count by 5s: 5, **10**, 15, 20, 25, . . .

Count by 10s: 10, 20, 30, 40, . . .

Note that **10** is a multiple of 1, 2, 5, and 10. A number is a multiple of all of its factors.

A **common multiple** is a multiple of two or more numbers. So, 10 is a common multiple of 1, 2, 5, and 10.

1. Multiply to list the next five multiples of 3.

3_____

2. Multiply to list the next five multiples of 7.

7

Is the number a factor of 8? Write yes or no.

3. 2

4. 8

5. 15

6. 20

Is the number a multiple of 4? Write yes or no.

7. 2

8. 12

9. 16

10. 18

Prime and Composite Numbers

A **prime number** is a whole number greater than 1 that has exactly two factors, 1 and the number itself.

A **composite number** is a whole number greater than 1 that has more than two factors.

You can use division to find the factors of a number and tell whether the number is prime or composite.

Tell whether 55 is prime or composite.

Use division to find all the numbers that divide into 55 without a remainder. Those numbers are the factors of 55.

$$55 \div 1 = 55$$
, so 1 and 55 are factors.

$$55 \div 5 = 11$$
, so $\underline{5}$ and $\underline{11}$ are factors.

The factors of 55 are $\frac{1}{5}$, $\frac{5}{5}$, $\frac{11}{5}$, and $\frac{55}{5}$.

Because 55 has more than two factors, 55 is a composite number.

Tell whether 61 is prime or composite.

Use division to find all the numbers that divide into 61 without a remainder. Those numbers are the factors of 61.

$$61 \div 1 = 61$$
, so $\underline{1}$ and $\underline{61}$ are factors.

There are no other numbers that divide into 61 evenly without a remainder.

The factors of 61 are $\frac{1}{2}$ and $\frac{61}{2}$.

Because 61 has exactly two factors, 61 is a prime number.

Tell whether the number is prime or composite.

- Think: Is 44 divisible by any number other than 1 and 44?
- **2.** 53

Think: Does 53 have other factors besides 1 and itself?

3. 12

4. 50

5. 24

6. 67

7. 83

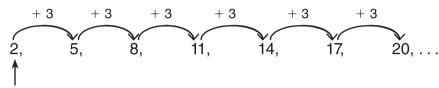
8. 27

9. 34

10. 78

Algebra • Number Patterns

A pattern is an ordered set of numbers or objects, called terms. The numbers below form a pattern. The first term in the pattern is 2.

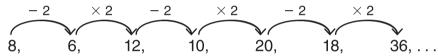


First term

A rule is used to describe a pattern. The rule for this pattern is add 3.

You can describe other patterns in the numbers. Notice that the terms in the pattern shown alternate between even and odd numbers.

For some patterns, the rule may have two operations.



The rule for this pattern is subtract 2, multiply by 2. The first term is 8. Notice that all of the terms in this pattern are even numbers.

Use the rule to write the numbers in the pattern.

- **1.** Rule: Add 7. First term: 12 **2.** Rule: Multiply by 3, subtract 1. First term: 2

Use the rule to write the numbers in the pattern. Describe another pattern in the numbers.

3. Rule: Subtract 5. First term: 50

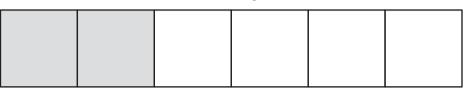
50, ____, ___, ___, ___, ...

4. Rule: Multiply by 2, add 1. First term: 4

Equivalent Fractions

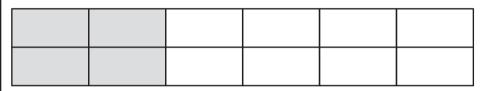
Write two fractions that are equivalent to $\frac{2}{6}$.

Step 1 Make a model to represent $\frac{2}{6}$.



The rectangle is divided into 6 equal parts, with 2 parts shaded.

Step 2 Divide the rectangle from Step 1 in half.



The rectangle is now divided into 12 equal parts, with 4 parts shaded.

The model shows the fraction $\frac{4}{12}$. So, $\frac{2}{6}$ and $\frac{4}{12}$ are equivalent.

Step 3 Draw the same rectangle as in Step 1, but with only 3 equal parts. Keep the same amount of the rectangle shaded.



The rectangle is now divided into 3 equal parts, with 1 part shaded.

The model shows the fraction $\frac{1}{3}$. So, $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent.

Use models to write two equivalent fractions.

1. $\frac{2}{4}$

2. $\frac{4}{6}$

Generate Equivalent Fractions

Write an equivalent fraction for $\frac{4}{5}$.

Step 1 Choose a whole number, like 2.

Step 2 Create a fraction using 2 as the numerator and denominator: $\frac{2}{2}$. This fraction is equal to 1. You can multiply a number by 1 without changing the value of the number.

Step 3 Multiply $\frac{4}{5}$ by $\frac{2}{2}$: $\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$.

So, $\frac{4}{5}$ and $\frac{8}{10}$ are equivalent.

Write another equivalent fraction for $\frac{4}{5}$.

Step 1 Choose a different whole number, like 20.

Step 2 Create a fraction using 20 as the numerator and denominator: $\frac{20}{20}$.

Step 3 Multiply $\frac{4}{5}$ by $\frac{20}{20}$: $\frac{4 \times 20}{5 \times 20} = \frac{80}{100}$.

So, $\frac{4}{5}$ and $\frac{80}{100}$ are equivalent.

Write two equivalent fractions.

1.
$$\frac{2}{6}$$

2.
$$\frac{4}{10}$$

3.
$$\frac{3}{8}$$

4.
$$\frac{3}{5}$$

Simplest Form

A fraction is in **simplest form** when 1 is the only factor that the numerator and denominator have in common.

Tell whether the fraction $\frac{7}{8}$ is in simplest form.

Look for common factors in the numerator and the denominator.

Step 1 The numerator of $\frac{7}{8}$ is 7. List all the	$1 \times 7 = 7$	
factors of 7.	The factors of 7 are 1 and 7.	
Step 2 The denominator of $\frac{7}{8}$ is 8. List all the factors of 8.	$1 \times 8 = 8$ $2 \times 4 = 8$	
	The factors of 8 are 1, 2, 4, and 8.	
Step 3 Check if the numerator and	The only common factor of 7 and 8 is 1.	
denominator of $\frac{7}{8}$ have any common		
factors greater than 1.		
So 7 is in simplest form		

Tell whether the fraction is in simplest form. Write yes or no.

1.
$$\frac{4}{10}$$

2.
$$\frac{2}{8}$$

3.
$$\frac{3}{5}$$

Write the fraction in simplest form.

4.
$$\frac{4}{12}$$

5.
$$\frac{6}{10}$$

6.
$$\frac{3}{6}$$

Common Denominators

A **common denominator** is a common multiple of the denominators of two or more fractions.

Write $\frac{2}{3}$ and $\frac{3}{4}$ as a pair of fractions with common denominators.

Step 1 Identify the denominators of $\frac{2}{3}$ and $\frac{3}{4}$.	$\frac{2}{3}$ and $\frac{3}{4}$ The denominators are 3 and 4.
Step 2 List multiples of 3 and 4. Circle common multiples.	3: 3, 6, 9, 12, 15, 18 4: 4, 8, 12, 16, 20 12 is a common multiple of 3 and 4.
Step 3 Rewrite $\frac{2}{3}$ as a fraction with a denominator of 12.	$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$
Step 4 Rewrite $\frac{3}{4}$ as a fraction with a denominator of 12.	$\frac{3}{4} = \frac{3 \times 3}{4 \times \underline{3}} = \frac{9}{12}$
So, you can rewrite $\frac{2}{3}$ and $\frac{3}{4}$ as $\frac{8}{12}$ and $\frac{9}{12}$.	

Write the pair of fractions as a pair of fractions with a common denominator.

1.
$$\frac{1}{2}$$
 and $\frac{1}{3}$

2.
$$\frac{2}{4}$$
 and $\frac{5}{8}$

3.
$$\frac{1}{2}$$
 and $\frac{3}{5}$

4.
$$\frac{1}{4}$$
 and $\frac{5}{6}$

5.
$$\frac{2}{5}$$
 and $\frac{2}{3}$

6.
$$\frac{4}{5}$$
 and $\frac{7}{10}$

Problem Solving • Find Equivalent Fractions

Kyle's mom bought bunches of balloons for a family party. Each bunch has 4 balloons, and $\frac{1}{4}$ of the balloons are blue. If Kyle's mom bought 5 bunches of balloons, how many balloons did she buy? How many of the balloons are blue?

Read the Problem						
I need to find how many balloons Kyle's mom bought and how many of the balloons are blue.	What information do I need to use? Each bunch has 1 out of 4 balloons that are blue, and there are 5 bunches.	I will make a table to find the total number balloons Kyle's mom bought and the fraction of balloons that are blue.				
Solve the Problem						
I can make a table						

I can make a table.

Number of Bunches	1	2	3	4	5
Total Number of Blue Balloons	1	2	3	4	5
Total Number of Balloons	4	8	12	16	20

Kyle's mom bought 20 balloons. 5 of the balloons are blue.

Make a table to solve.

- 1. Jackie is making a beaded bracelet. The bracelet will have no more than 12 beads. ¹/₃ of the beads on the bracelet will be green. What other fractions could represent the part of the beads on the bracelet that will be green?
- 2. Ben works in his dad's bakery packing bagels. Each package can have no more than 16 bagels. 3/4 of the bagels in each package are plain. What other fractions could represent the part of the bagels in each package that will be plain?

Compare Fractions Using Benchmarks

A **benchmark** is a known size or amount that helps you understand a different size or amount. You can use $\frac{1}{2}$ as a benchmark.

Sara reads for $\frac{3}{6}$ hour every day after school. Connor reads for $\frac{2}{3}$ hour. Who reads for a longer amount of time?

Compare the fractions. $\frac{3}{6} \bigcirc \frac{2}{3}$

$$\frac{3}{6}$$
 $\frac{2}{3}$

Step 1 Divide one circle into 6 equal parts. Divide another circle into 3 equal parts.

Step 2 Shade $\frac{3}{6}$ of the first circle. How many parts will you shade? 3 parts

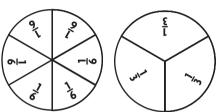
Step 3 Shade $\frac{2}{3}$ of the second circle.

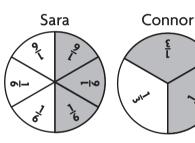
How many parts will you shade? 2 parts

Step 4 Compare the shaded parts of each circle. Half of Sara's circle is shaded. More than half of Connor's circle is shaded.

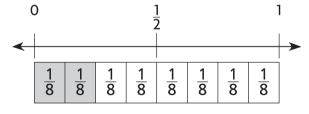


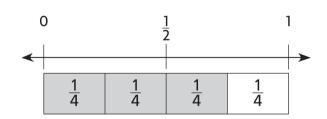
So, Connor reads for a longer amount of time.





1. Compare $\frac{2}{8}$ and $\frac{3}{4}$. Write < or >.







R51

Compare. Write < or >.



3.
$$\frac{7}{8}$$
 $\frac{1}{3}$

4.
$$\frac{5}{12}$$
 $\frac{1}{2}$

6.
$$\frac{4}{6}$$
 $\frac{4}{8}$

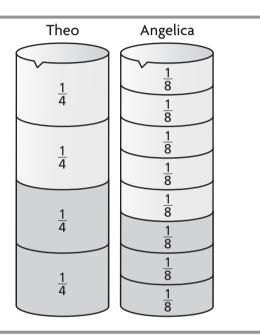
7.
$$\frac{7}{12}$$
 $\frac{2}{4}$

Compare Fractions

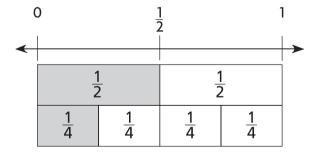
Theo filled a beaker $\frac{2}{4}$ full with water. Angelica filled a beaker $\frac{3}{8}$ full with water. Whose beaker has more water?

Compare $\frac{2}{4}$ and $\frac{3}{8}$.

- Step 1 Divide one beaker into 4 equal parts. Divide another beaker into 8 equal parts.
- **Step 2** Shade $\frac{2}{4}$ of the first beaker.
- **Step 3** Shade $\frac{3}{8}$ of the second beaker.
- Step 4 Compare the shaded parts of each beaker. Half of Theo's beaker is shaded. Less than half of Angelica's beaker is shaded.
- is greater than $\frac{\overline{8}}{}$.
- So, Theo's beaker has more water.

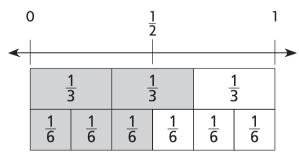


1. Compare $\frac{1}{2}$ and $\frac{1}{4}$.



Which is greater? _____

2. Compare $\frac{2}{3}$ and $\frac{3}{6}$.



Which is less? _____

Compare. Write <, >, or =.

- 3. $\frac{1}{2}$ $\bigcirc \frac{3}{4}$ 4. $\frac{6}{12}$ $\bigcirc \frac{5}{8}$ 5. $\frac{2}{3}$ $\bigcirc \frac{4}{6}$ 6. $\frac{3}{8}$

Compare and Order Fractions

Write $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ in order from least to greatest.

Step 1 Identify a common denominator. Multiples of 8:(8,)16, 24

Multiples of 4: 4,(8,)16,

Multiples of 2: 2, 4, 6,(8)

Use 8 as a common denominator.

Step 2 Use the common denominator to write equivalent fractions.

 $\frac{\frac{3}{8}}{\frac{1}{4}} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$ $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

Step 3 Compare the numerators.

2 < 3 < 4

Step 4 Order the fractions from least to greatest, using < or > symbols.

 $\frac{2}{8} < \frac{3}{8} < \frac{4}{8}$

So,
$$\frac{1}{4} < \frac{3}{8} < \frac{1}{2}$$
.

Write the fraction with the greatest value.

1.
$$\frac{2}{3}$$
, $\frac{1}{4}$, $\frac{1}{6}$

2.
$$\frac{3}{10}$$
, $\frac{1}{2}$, $\frac{2}{5}$

3.
$$\frac{1}{8}$$
, $\frac{5}{12}$, $\frac{9}{10}$

Write the fractions in order from least to greatest.

4.
$$\frac{9}{10}$$
, $\frac{1}{2}$, $\frac{4}{5}$

5.
$$\frac{3}{4}$$
, $\frac{7}{8}$, $\frac{1}{2}$

6.
$$\frac{2}{3}$$
, $\frac{3}{4}$, $\frac{5}{6}$

Add and Subtract Parts of a Whole

Justin has $\frac{3}{8}$ pound of cheddar cheese and $\frac{2}{8}$ pound of brick cheese. How much cheese does he have in all?

- **Step 1** Use fraction strips to model the problem. Use three $\frac{1}{8}$ -strips to represent $\frac{3}{8}$ pound of cheddar cheese.
- **Step 2** Join two more $\frac{1}{8}$ -strips to represent the amount of brick cheese.

Step 4 Use the model to write an equation. $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$

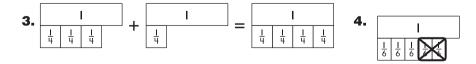
Suppose Justin eats $\frac{1}{8}$ pound of cheese. How much cheese is left?

- **Step 1** Use five $\frac{1}{8}$ -strips to represent the $\frac{5}{8}$ pound of cheese.
- **Step 2** Remove one $\frac{1}{8}$ -strip to show the amount eaten.
- Step 3 Count the number of $\frac{1}{8}$ -strips left. There are $\frac{1}{8}$ fraction strips. There is $\frac{4}{8}$ pound left.

Step 4 Write an equation for the model.

 $\frac{5}{8} - \frac{1}{8} = \frac{4}{8}$

Use the model to write an equation.



Write Fractions as Sums

A **unit fraction** tells the part of the whole that 1 piece represents.

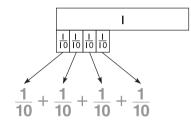
A unit fraction always has a numerator of 1.

Bryan has $\frac{4}{10}$ pound of clay for making clay figures. He wants to use $\frac{1}{10}$ pound of clay for each figure. How many clay figures can he make?

Use fraction strips to write $\frac{4}{10}$ as a sum of unit fractions.

Step 1 Represent $\frac{4}{10}$ with fraction strips.

Step 2 Each $\frac{1}{10}$ is a unit fraction. Write a $\frac{1}{10}$ addend for each $\frac{1}{10}$ -strip you used to show $\frac{4}{10}$.



Step 3 Count the number of addends. The number of addends represents the number of clay figures Bryan can make.

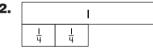
So, Bryan can make 4 clay figures.

Write the fraction as the sum of unit fractions.

1

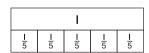
١.				I
	<u>l</u>	<u>1</u>	<u>l</u>	

2.



$$\frac{2}{4} =$$
_____ + ____





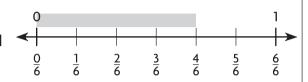
$$\frac{5}{5} =$$

Add Fractions Using Models

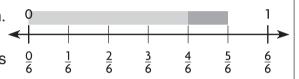
Fractions with like denominators have the same denominator. You can add fractions with like denominators using a number line.

Model $\frac{4}{6} + \frac{1}{6}$.

Step 1 Draw a number line labeled with sixths. Model the fraction $\frac{4}{6}$ by starting at 0 and shading 4 sixths.



Step 2 Add the fraction $\frac{1}{6}$ by shading 1 more sixth.



Step 3 How many sixths are there in all? 5 sixths Write the number of sixths as a fraction.

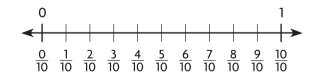
5 sixths =
$$\frac{5}{6}$$
 $\frac{4}{6} + \frac{1}{6} = \frac{5}{6}$

1. Model $\frac{1}{5} + \frac{4}{5}$. $\frac{1}{5} + \frac{4}{5} =$

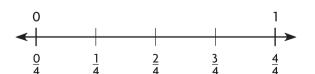


Find the sum. Use a model to help.







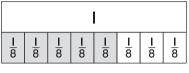


Subtract Fractions Using Models

You can subtract fractions with like denominators using fraction strips.

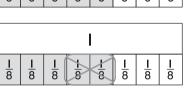
Model
$$\frac{5}{8} - \frac{2}{8}$$
.

Step 1 Shade the eighths you start with. Shade 5 eighths.



Step 2 Subtract $\frac{2}{8}$.

Think: How many eighths are taken away? Cross out 2 of the shaded eighths.

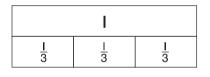


- **Step 3** Count the shaded eighths that remain. There are 3 eighths remaining.
- **Step 4** Write the number of eighths that remain as a fraction.

3 eighths
$$=\frac{3}{8}$$
 $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

1. Model $\frac{3}{3} - \frac{2}{3}$.



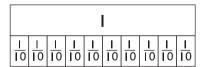


Subtract. Use fraction strips to help.

2. $\frac{5}{6} - \frac{1}{6}$

<u>I</u>	<u>I</u>	<u>I</u>	<u>I</u>	<u> </u> 6	<u>I</u>

3. $\frac{6}{10} - \frac{3}{10}$



$$\frac{5}{6} - \frac{1}{6} =$$

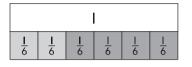
$$\frac{6}{10} - \frac{3}{10} =$$

Add and Subtract Fractions

You can find and record the sums and the differences of fractions.

Add.
$$\frac{2}{6} + \frac{4}{6}$$

Step 1 Model it.



Step 2 Think: How many sixths are there in all?

There are 6 sixths.

6 sixths =
$$\frac{6}{6}$$

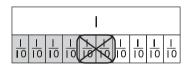
Step 3 Record it.

Write the sum as an addition equation.

$$\frac{2}{6} + \frac{4}{6} = \frac{6}{6}$$

Subtract.
$$\frac{6}{10} - \frac{2}{10}$$

Step 1 Model it.



Step 2 Think: There are 6 tenths. I take away 2 tenths. How many tenths are left?

There are 4 tenths left.

4 tenths =
$$\frac{4}{10}$$

Step 3 Record it.

Write the difference as a subtraction equation.

$$\frac{6}{10} - \frac{2}{10} = \frac{4}{10}$$

Find the sum or difference.

1. 7 eighth-size parts — 4 eighth-size parts = _____

$$\frac{7}{8} - \frac{4}{8} =$$

2.
$$\frac{11}{12} - \frac{4}{12} = \underline{\hspace{1cm}}$$

2.
$$\frac{11}{12} - \frac{4}{12} =$$
 3. $\frac{2}{10} + \frac{2}{10} =$ **4.** $\frac{6}{8} - \frac{4}{8} =$ **4.**

4.
$$\frac{6}{8} - \frac{4}{8} =$$

5.
$$\frac{2}{4} + \frac{2}{4} =$$

6.
$$\frac{4}{5} - \frac{3}{5} =$$

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5.
$$\frac{2}{4} + \frac{2}{4} =$$
 7. $\frac{1}{3} + \frac{2}{3} =$

Rename Fractions and Mixed Numbers

A **mixed number** is made up of a whole number and a fraction. You can use multiplication and addition to rename a mixed number as a fraction greater than 1.

Rename $2\frac{5}{6}$ as a fraction.

First, multiply the denominator, or the number of parts in the whole, by the whole number.

$$2\frac{5}{6} = \frac{17}{6}$$
 of parts number

total number number of parts in the whole

$$6 \times 2 = 12$$

Then, add the numerator to your product.

$$12 + 5 = 17$$

So,
$$2\frac{5}{6} = \frac{17}{6}$$
.

You can use division to write a fraction greater than 1 as a mixed number.

Rename $\frac{16}{3}$ as a mixed number.

To rename $\frac{16}{3}$ as a mixed number, divide the numerator by the denominator.

Use the quotient and remainder to write a mixed number.

So,
$$\frac{16}{3} = 5\frac{1}{3}$$
.

Write the mixed number as a fraction.

1.
$$3\frac{2}{3} =$$

2.
$$4\frac{3}{5} =$$

3.
$$4\frac{3}{8} =$$

1.
$$3\frac{2}{3} =$$
 2. $4\frac{3}{5} =$ **3.** $4\frac{3}{8} =$ **4.** $2\frac{1}{6} =$

Write the fraction as a mixed number.

5.
$$\frac{32}{5} =$$

6.
$$\frac{19}{3} =$$

7.
$$\frac{15}{4} =$$

5.
$$\frac{32}{5} =$$
 6. $\frac{19}{3} =$ **7.** $\frac{15}{4} =$ **8.** $\frac{51}{10} =$ ____

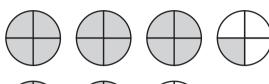
Add and Subtract **Mixed Numbers**

Find the sum. $3\frac{1}{4} + 2\frac{1}{4}$

Add the whole number and fraction parts.

- Add the whole numbers: 3 + 2 = 5• Add the fractions: $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

Write the sum as a mixed number, so the fractional part is less than 1. $3\frac{1}{4} + 2\frac{1}{4} = 5\frac{2}{4}$



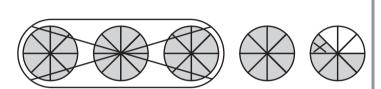
Find the difference. $4\frac{5}{8} - 3\frac{1}{8}$

Subtract the fraction and the whole number parts.

- Subtract the fractions: $\frac{5}{8} \frac{1}{8} = \frac{4}{8}$
- Subtract the whole numbers:

$$4 - 3 = 1$$

$$4\frac{5}{8} - 3\frac{1}{8} = 1\frac{4}{8}$$



Find the sum or difference.

1.
$$3\frac{4}{5}$$

$$+4\frac{3}{5}$$

2.
$$7\frac{2}{3}$$

$$-3\frac{1}{3}$$

3.
$$4\frac{7}{12}$$

$$+6\frac{5}{12}$$

4.
$$12\frac{3}{4}$$

$$-6\frac{1}{4}$$

5.
$$2\frac{3}{8}$$

$$+ 8\frac{1}{8}$$

6.
$$11\frac{9}{10}$$

$$-3\frac{7}{10}$$

7.
$$7\frac{3}{5}$$

$$+4\frac{3}{5}$$

8.
$$8\frac{3}{6}$$

$$-3\frac{1}{6}$$

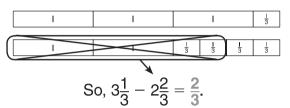
Subtraction with Renaming

Fraction strips can help you subtract mixed numbers or subtract a mixed number from a whole number.

Find the difference. $3\frac{1}{3} - 2\frac{2}{3}$

Step 1 Model the number you are subtracting from, $3\frac{1}{3}$.

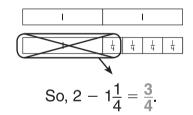
Step 2 Because you cannot subtract $\frac{2}{3}$ from $\frac{1}{3}$ without renaming, change one of the 1 strips to three $\frac{1}{3}$ strips. Then subtract by crossing out two wholes and two $\frac{1}{3}$ strips.



Find the difference. $2 - 1\frac{1}{4}$

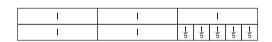
Step 1 Model the number you are subtracting from, 2.

Step 2 Because you cannot subtract $\frac{1}{4}$ from 1 without renaming, change one of the 1 strips to four $\frac{1}{4}$ strips. Then subtract by crossing out one whole and one $\frac{1}{4}$ strip.



Find the difference.

1.
$$3-2\frac{2}{5}=$$



2.
$$2\frac{1}{4} - 1\frac{3}{4} =$$



3.
$$3\frac{3}{5}$$
 $-2\frac{4}{5}$

4.
$$3\frac{1}{12}$$
 $-2\frac{11}{12}$

5.
$$4\frac{5}{8}$$
 $-2\frac{7}{8}$

Fractions and Properties of Addition

Properties of addition can help you group and order addends so you can use mental math to find sums.

The Commutative Property of Addition states that when the order of two addends is changed, the sum is the same.

$$6 + 3 = 3 + 6$$

The Associative Property of Addition states that when the grouping of addends is changed, the sum is the same.

$$(3+6)+4=3+(6+4)$$

Use the properties and mental math to add $10\frac{3}{8} + 4\frac{7}{8} + 6\frac{5}{8}$.

Step 1 Look for fractions that combine to make 1.

$$10\frac{3}{8} + 4\frac{7}{8} + 6\frac{5}{8}$$

Step 2 Use the Commutative Property to order the addends so that the fractions with a sum of 1 are together.

$$10\frac{3}{8} + 4\frac{7}{8} + 6\frac{5}{8} = 10\frac{3}{8} + 6\frac{5}{8} + 4\frac{7}{8}$$

Step 3 Use the Associative Property to group the addends that you can add mentally.

$$= \left(10\frac{3}{8} + 6\frac{5}{8}\right) + 4\frac{7}{8}$$

Step 4 Add the grouped numbers and then add the other mixed number.

$$= (17) + 4\frac{7}{8}$$

Step 5 Write the sum.

$$=21\frac{7}{8}$$

Use the properties and mental math to find the sum.

1.
$$\left(3\frac{1}{5}+1\frac{2}{5}\right)+4\frac{4}{5}$$

1.
$$\left(3\frac{1}{5}+1\frac{2}{5}\right)+4\frac{4}{5}$$
 2. $\left(5\frac{7}{10}+1\frac{4}{10}\right)+6\frac{3}{10}$ **3.** $7\frac{3}{4}+\left(5+3\frac{1}{4}\right)$

3.
$$7\frac{3}{4} + \left(5 + 3\frac{1}{4}\right)$$

4.
$$\left(2\frac{5}{12}+3\frac{11}{12}\right)+1\frac{7}{12}$$
 5. $4\frac{7}{8}+\left(6\frac{3}{8}+\frac{1}{8}\right)$ **6.** $9\frac{2}{6}+\left(4\frac{1}{6}+7\frac{4}{6}\right)$

5.
$$4\frac{7}{8} + \left(6\frac{3}{8} + \frac{1}{8}\right)$$

6.
$$9\frac{2}{6} + \left(4\frac{1}{6} + 7\frac{4}{6}\right)$$

Problem Solving • Multistep Fraction Problems

Jeff runs $\frac{3}{5}$ mile each day. He wants to know how many days he has to run before he has run a whole number of miles.

Read the Problem

What do I need to find?

I need to find how many days Jeff needs to run $\frac{3}{5}$ mile until he has

run a whole number of miles.

What information do I need to use?

Jeff runs ______ mile a day. He wants the distance run to be a whole number ____

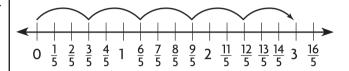
How will I use the information?

I can use a number line and

patterns to act out
the problem.

Solve the Problem

Describe how to act it out. Use a number line.



Day 1: $\frac{3}{5}$ mile Day 2: $\frac{6}{5}$ mile $\frac{3}{5}$ + $\frac{3}{5}$ = $\frac{6}{5}$

1 whole mile and $\frac{1}{5}$ mile more

Day 3: $\frac{9}{5}$ mile $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{9}{5}$

1 whole mile and $\frac{4}{5}$ mile more

Day 4: $\frac{12}{5}$ mile $\frac{\frac{3}{5}}{5}$ + $\frac{\frac{3}{5}}{5}$ + $\frac{\frac{3}{5}}{5}$ = $\frac{12}{5}$

2 whole miles and $\frac{2}{5}$ mile more

Day 5: $\frac{15}{5}$ mile $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{15}{5}$

3 whole miles

So, Jeff will run a total of 3 miles in 5 days.

- **1.** Lena runs $\frac{2}{3}$ mile each day. She wants to know how many days she has to run before she has run a whole number of miles.
- **2.** Mack is repackaging $\frac{6}{8}$ -pound bags of birdseed into 1-pound bags of birdseed. What is the least number of $\frac{6}{8}$ -pound bags of birdseed he needs in order to fill 1-pound bags without leftovers?

Multiples of Unit Fractions

A unit fraction is a fraction with a numerator of 1. You can write a fraction as the product of a whole number and a unit fraction.

Write $\frac{7}{10}$ as the product of a whole number and a unit fraction.

Write $\frac{7}{10}$ as the sum of unit fractions.

$$\frac{7}{10} = \frac{1}{10} + \frac{1}{10}$$

Use multiplication to show repeated addition.

$$\frac{7}{10} = \underline{7} \times \frac{1}{10}$$

So,
$$\frac{7}{10} = \frac{7}{10} \times \frac{1}{10}$$

The product of a number and a counting number is a multiple of the number. You can find multiples of unit fractions.

List the next 4 multiples of $\frac{1}{8}$.

Make a table and use repeated addition.

$1 \times \frac{1}{8}$	$2 \times \frac{1}{8}$	$3 \times \frac{1}{8}$	$4 \times \frac{1}{8}$	$5 \times \frac{1}{8}$
<u>1</u> 8	$\frac{1}{8} + \frac{1}{8}$	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$	$\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$
<u>1</u> 8	<u>2</u> 8	38	48	<u>5</u> 8

The next 4 multiples of $\frac{1}{8}$ are $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$ and $\frac{5}{8}$

Write the fraction as the product of a whole number and a unit fraction.

1.
$$\frac{2}{5} =$$

1.
$$\frac{2}{5} =$$
 2. $\frac{5}{12} =$

3.
$$\frac{7}{2} =$$

List the next four multiples of the unit fraction.

Multiples of Fractions

You have learned to write multiples of unit fractions. You can also write multiples of other fractions.

Write the next 4 multiples of $\frac{2}{5}$.

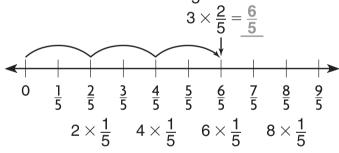
Make a table.

$1 \times \frac{2}{5}$	$2 imes \frac{2}{5}$	$3 \times \frac{2}{5}$	$4 \times \frac{2}{5}$	$5 \times \frac{2}{5}$
<u>2</u> 5	$\frac{2}{5} + \frac{2}{5}$	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5}$	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$	$\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$
<u>2</u> 5	<u>4</u> <u>5</u>	<u>6</u> <u>5</u>	<u>8</u> <u>5</u>	<u>10</u> <u>5</u>

So, the next 4 multiples of $\frac{2}{5}$ are $\frac{4}{5}$, $\frac{6}{5}$, $\frac{8}{5}$, and $\frac{10}{5}$.

Write $3 \times \frac{2}{5}$ as the product of a whole number and a unit fraction.

Use a number line. Make three jumps of $\frac{2}{5}$.



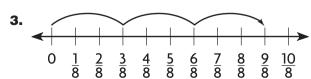
So, $3 \times \frac{2}{5} = \frac{6}{5}$, or $6 \times \frac{1}{5}$.

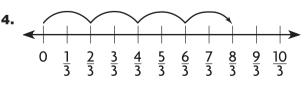
List the next four multiples of the fraction.

1.
$$\frac{3}{4}$$
, _____, ____, ____

2.
$$\frac{5}{6}$$
, _____, ____, _____

Write as the product of a whole number and a unit fraction.





$$3 \times \frac{3}{8} =$$

$$4 \times \frac{2}{3} =$$

Multiply a Fraction by a **Whole Number Using Models**

You can use a model to multiply a fraction by a whole number.

Find the product of $4 \times \frac{3}{5}$.

Use fraction strips. Show 4 groups of $\frac{3}{5}$ each.

1 group of
$$\frac{3}{5} = \frac{3}{5}$$

2 groups of
$$\frac{3}{5} = \frac{6}{5}$$

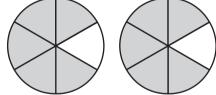
3 groups of
$$\frac{3}{5} = \frac{9}{5}$$

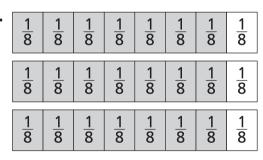
4 groups of
$$\frac{3}{5} = \frac{12}{5}$$

So,
$$4 \times \frac{3}{5} = \frac{12}{5}$$
.

Multiply.







$$2 \times \frac{5}{6} =$$

$$3 \times \frac{7}{8} =$$

3.
$$6 \times \frac{2}{3} =$$

4.
$$2 \times \frac{9}{10} =$$
 5. $5 \times \frac{3}{4} =$

5.
$$5 \times \frac{3}{4} =$$

6.
$$4 \times \frac{5}{8} =$$

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7.
$$7 \times \frac{2}{5} =$$

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8.
$$8 \times \frac{4}{6} =$$

Multiply a Fraction or Mixed Number by a Whole Number

To multiply a fraction by a whole number, multiply the numerators. Then multiply the denominators.

A recipe for one loaf of bread calls for $2\frac{1}{4}$ cups of flour. How many cups of flour will you need for 2 loaves of bread?

Step 1 Write and solve an equation.

$$2 \times 2\frac{1}{4} = \frac{2}{1} \times \frac{9}{4}$$
 Write 2 as $\frac{2}{1}$. Write $2\frac{1}{4}$ as a fraction.
$$= \frac{2 \times 9}{1 \times 4}$$
 Multiply the numerators. Then multiply the denominators.
$$= \frac{18}{4}$$
 Simplify.

Step 2 Write the product as a mixed number.

$$\frac{18}{4} = \underbrace{\frac{1}{4} + \frac{1}{4} + \frac$$

Multiply. Write the product as a mixed number.

1.
$$3 \times \frac{2}{5} =$$
 2. $4 \times \frac{3}{8} =$

2.
$$4 \times \frac{3}{8} =$$

3.
$$5 \times \frac{1}{3} =$$

4.
$$2 \times 1\frac{3}{10} =$$
 5. $4 \times 1\frac{2}{3} =$

5.
$$4 \times 1\frac{2}{3} =$$

6.
$$7 \times 1\frac{1}{6} =$$

Problem Solving • Comparison Problems with Fractions

The Great Salt Lake in Utah is about $\frac{4}{5}$ mile above sea level. Lake Titicaca in South America is about 3 times as high above sea level as the Great Salt Lake. About how high above sea level is Lake Titicaca?

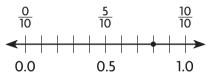
Read the Problem	Solve the Problem
What do I need to find? I need to find about how high above sea level Lake Titicaca is.	Draw a comparison model. Compare the heights above sea level of the Great Salt Lake and Lake Titicaca, in miles. Great Salt Lake 4 5
What information do I need to use? The Great Salt Lake is about mile above sea level. Lake Titicaca is about times as high above sea level.	Lake Titicaca 4 5 5 5 t
	Write an equation and solve.
How will I use the information?	t is the height above sea level of Lake Titicaca, in miles.
I can draw a diagram to compare the	$t = \frac{3}{12} \times \frac{\frac{1}{5}}{5}$ Write an equation.
heights.	$t = \frac{12}{5}$ Multiply.
	$t = \frac{2\frac{2}{5}}{}$ Write the fraction as a mixed number.
So, Lake Titicaca is about miles abov	/e sea level.

- **1.** Amelia is training for a triathlon. She swims $\frac{3}{5}$ mile. Then she runs about 6 times farther than she swims. About how far does Amelia run?
- 2. Last week, Meg bought $1\frac{3}{4}$ pounds of fruit at the market. This week, she buys 4 times as many pounds of fruit as last week. In pounds, how much fruit does Meg buy this week?

Relate Tenths and Decimals

Write the fraction and the decimal that are shown by the point

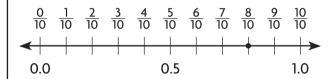
on the number line.



Step 1 Count the number of equal parts of the whole shown on the number line. There are ten equal parts.

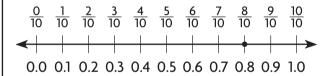
This tells you that the number line shows tenths.

Step 2 Label the number line with the missing fractions. What fraction is shown by the point on the number line?



The fraction shown by the point on the number line is $\frac{8}{10}$.

Step 3 Label the number line with the missing decimals. What decimal is shown by the point on the number line?

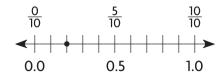


The decimal shown by the point on the number line is **0.8**.

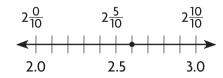
So, the fraction and decimal shown by the point on the number line are $\frac{8}{10}$ and 0.8.

Write the fraction or mixed number and the decimal shown by the model.

1.



2

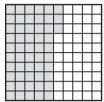


Lesson 9.2

Reteach

Relate Hundredths and Decimals

Write the fraction or mixed number and the decimal shown by the model.

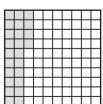


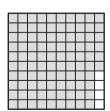
ı		
	Step 1 Count the number of shaded squares in the model and the total number of squares in the whole model.	Number of shaded squares: 53 Total number of squares: 100
	Step 2 Write a fraction to represent the part of the model that is shaded.	$\frac{\text{Number of Shaded Squares}}{\text{Total Number of Squares}} = \frac{53}{100}$ The fraction shown by the model is $\frac{53}{100}$.
	Step 3 Write the fraction in decimal form.	Think: The fraction shown by the model is $\frac{53}{100}$.
		0.53 names the same amount as $\frac{53}{100}$.
		The decimal shown by the model is 0.53.

The fraction and decimal shown by the model are $\frac{53}{100}$ and 0.53.

Write the fraction or mixed number and the decimal shown by the model.

1.





Equivalent Fractions and Decimals

Lori ran $\frac{20}{100}$ mile. How many tenths of a mile did she run?

Write $\frac{20}{100}$ as an equivalent fraction with a denominator of 10.

Step 1 Think: 10 is a common factor of the numerator and the denominator.

Step 2 Divide the numerator and denominator by 10.

$$\frac{20}{100} = \frac{20 \div 10}{100 \div 10} = \frac{2}{10}$$

So, Lori ran $\frac{2}{10}$ mile.

Use a place-value chart.

Step 1 Write $\frac{20}{100}$ as an equivalent decimal.

Ones	•	Tenths	Hundredths
0	-	2	0

Step 2 Think: 20 hundredths is 2 tenths 0 hundredths

Ones	•	Tenths
0		2

So, Lori ran 0.2 mile.

Write the number as hundredths in fraction form and decimal form.

1. $\frac{9}{10}$

2. 0.6

3. $\frac{4}{10}$

Write the number as tenths in fraction form and decimal form.

4. $\frac{70}{100}$

5. $\frac{80}{100}$

6. 0.50

Relate Fractions, Decimals, and Money

Write the total money amount. Then write the amount as a fraction and as a decimal in terms of a dollar.



Step 1 Count the value of coins from greatest to least. Write the total money amount.



Step 2 Write the total money amount as a fraction of a dollar.

The total money amount is \$0.50, which is the same as 50 cents.

Think: There are 100 cents in a dollar.

So, the total amount written as a fraction of a dollar is:

 $\frac{50 \text{ cents}}{100 \text{ cents}} = \frac{50}{100}$

Step 3 Write the total money amount as a decimal.

Think: I can write \$0.50 as **0.50**.

The total money amount is $\frac{50}{100}$ written as a fraction of a dollar, and 0.50 written as a decimal.

Write the total money amount. Then write the amount as a fraction or a mixed number and as a decimal in terms of a dollar.

1.



2









Problem Solving • Money

Use the strategy act it out to solve the problem.

Jessica, Brian, and Grace earned \$7.50. They want to share the money equally. How much will each person get?

Read the Problem	Solve the Problem
What do I need to find?	Show the total amount, \$7.50, using one-dollar bills and 2 quarters.
I need to find the amount of money each person should get	
What information do I need to use?	Share the one-dollar bills equally.
I need to use the total amount, <u>\$7.50</u> , and divide it by <u>3</u> , the number of people sharing the money equally.	There is1_ one-dollar bill left.
How will I use the information?	Change the dollar bill that is left for4_ quarters. Now there are6_ quarters.
I will use dollar bills and coins to model the total amount and	Share the quarters equally.
act out the problem	
	So, each person gets 2 one-dollar bills and 2 quarters, or \$2.50.

- **1.** Jacob, Dan, and Nathan were given \$6.90 to share equally. How much money will each boy get?
- 2. Becky, Marlis, and Hallie each earned \$2.15 raking leaves. How much did they earn together?

Add Fractional Parts of 10 and 100

Sam uses 100 glass beads for a project. Of the beads, $\frac{35}{100}$ are gold and $\frac{4}{10}$ are silver. What fraction of the glass beads are gold or silver?

Add $\frac{35}{100}$ and $\frac{4}{10}$.

Step 1 Decide on a common denominator. Use 100.

Step 2 Write $\frac{4}{10}$ as an equivalent fraction with a denominator of 100.

$$\frac{4}{10} = \frac{4 \times 10}{10 \times 10} = \frac{40}{100}$$

Step 3 Add $\frac{35}{100}$ and $\frac{40}{100}$.

$$\frac{35}{100} + \frac{40}{100} = \frac{75}{100}$$
 Add the numerators.

Use 100 as the denominator.

So, 100 of the glass beads are gold or silver.

Add \$0.26 and \$0.59.

Step 1 Write each amount as a fraction of a dollar.

$$$0.26 = \frac{26}{100} \text{ of a dollar}$$
 $$0.59 = \frac{59}{100} \text{ of a dollar}$

$$$0.59 = \frac{59}{100}$$
 of a dollar

Step 2 Add $\frac{26}{100}$ and $\frac{59}{100}$.

$$\frac{26}{100} + \frac{59}{100} = \frac{85}{100}$$

 $\frac{26}{100} + \frac{59}{100} = \frac{85}{100}$ Add the numerators. \leftarrow 100 is the common denominator.

Step 3 Write the sum as a decimal.

$$\frac{85}{100} = 0.85$$

So,
$$\$0.26 + \$0.59 = \$0.85$$

Find the sum.

1.
$$\frac{75}{100} + \frac{2}{10} = \frac{1}{10}$$

Compare Decimals

Alfie found 0.2 of a dollar and Gemma found 0.23 of a dollar. Which friend found more money?

To compare decimals, you can use a number line.

Step 1 Locate each decimal on a number line.



Step 2 The number farther to the right is greater.

0.23 > 0.2, so Gemma found more money.

To compare decimals, you can compare equal-size parts.

Step 1 Write 0.2 as a decimal in hundredths.

0.2 is 2 tenths, which is equivalent to 20 hundredths.

$$0.2 = 0.20$$

Step 2 Compare.

23 hundredths is greater than 20 hundredths, so 0.23 > 0.2.

So, Gemma found more money.

Compare. Write <, >, or =.

Lines, Rays, and Angles

Name	What it looks like	Think
point D	D•	A point names a location in space.
line AB ; \overrightarrow{AB} line BA ; \overrightarrow{BA}	A B	A line extends without end in opposite directions.
line segment AB ; \overline{AB} line segment BA ; \overline{BA}	A B	"Segment" means part. A line segment is part of a line. It is named by its two endpoints.
ray <i>MN; MN</i> ray <i>NM; NM</i>	M N	A ray has one endpoint and extends without end in one direction. A ray is named using two points. The endpoint is always named first.
angle XYZ ; $\angle XYZ$ angle ZYX ; $\angle ZYX$ angle Y ; $\angle Y$	X	Two rays or line segments that share an endpoint form an angle. The shared point is the vertex of the angle.
A right angle forms a square corner.	An acute angle opens less than a right angle.	An obtuse angle opens more than a right angle and less than a straight angle. A straight angle forms a line.

Draw and label an example of the figure.

1. \overline{PQ}

2. *KJ*

3. obtuse ∠*FGH*

Classify Triangles

A **triangle** is a polygon with ___3__ sides

and 3 angles.

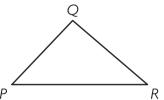
Each pair of sides joins at a vertex.

You can name a triangle by its vertices.

 ΔPOR ΔPRQ ΔQRP ΔQPR

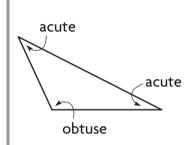
 ΔRPQ



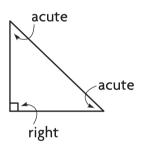


There are _____ types of triangles. All triangles have at least ____ acute angles.

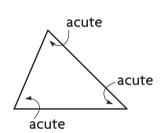
Obtuse triangle one obtuse angle



Right triangle one right angle



Acute triangle three acute angles

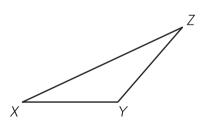


1. Name the triangle. Tell whether each angle is acute, right, or obtuse. A name for the triangle

∠*X* is ______

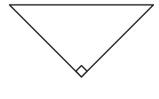
∠*Y* is _____

 $\angle Z$ is ______



Classify each triangle. Write acute, right, or obtuse.

2.







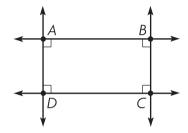
Parallel Lines and Perpendicular Lines

Parallel lines are lines in a plane that are always the same distance apart. Parallel lines or line segments never meet.

In the figure, lines AB and CD, even if extended, will never meet.

The lines are parallel. Write $\overrightarrow{AB} \parallel \overrightarrow{CD}$.

Lines AD and BC are also parallel. So, $\overrightarrow{AD} \overrightarrow{BC}$.



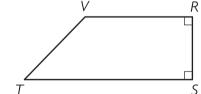
Intersecting lines cross at exactly one point. Intersecting lines that form right angles are **perpendicular**.

In the figure, lines AD and AB are perpendicular because they form right angles at vertex A. Write $\overrightarrow{AD} \perp \overrightarrow{AB}$.

Lines \underline{BC} and \underline{CD} are also perpendicular. So, $\overrightarrow{BC}\bot\overrightarrow{CD}$.

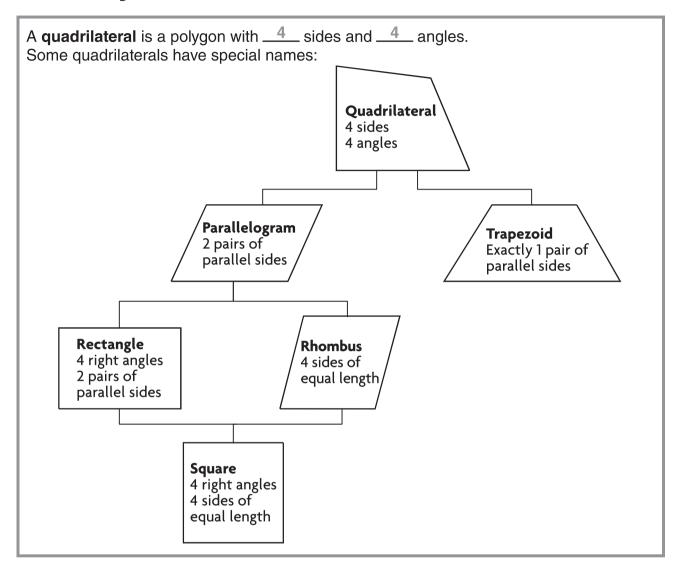
Use the figure for 1–3.

1. Name two sides that appear to be parallel.



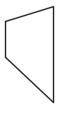
- 2. Name two sides that appear to be perpendicular.
- **3.** Name two sides that appear to be intersecting, but not perpendicular.

Classify Quadrilaterals



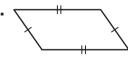
Classify each figure as many ways as possible. Write quadrilateral, trapezoid, parallelogram, rhombus, rectangle, or square.

1.



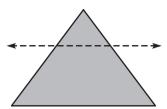
2.





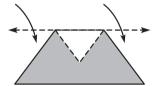
Line Symmetry

Tell whether the parts on each side of the line match. Is the line a line of symmetry?



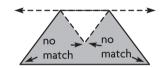
Step 1 Trace and cut out the shape.

Fold the shape along the dashed line.



Step 2 Tell whether the parts on each side match.

Compare the parts on each side.



The parts do not match.

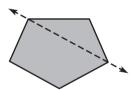
Step 3 Decide if the line is a line of symmetry.

The parts on each side of the line do not match.

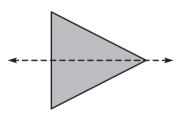
So, the line is not a line of symmetry.

Tell if the line appears to be a line of symmetry. Write yes or no.

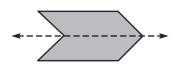
1.

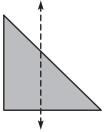


2.



3.





Find and Draw Lines of Symmetry

Tell whether the shape appears to have zero lines, 1 line, or more than 1 line of symmetry. Write zero, 1, or more than 1.



Step 1 Decide if the shape has a line of symmetry.

Trace and cut out the shape. Fold the shape along a vertical line.



Do the two parts match exactly? <u>yes</u>

Step 2 Decide if the shape has another line of symmetry.

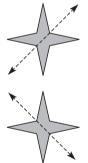
Open the shape and fold it along a horizontal line.



Do the two parts match exactly? <u>yes</u>

Step 3 Find any other lines of symmetry.

Think: Can I fold the shape in other ways so that the two parts match exactly?



I can fold the paper diagonally two different ways, and the parts match exactly.

So, the shape appears to have _____ more than 1

_ line of symmetry.

Tell whether the shape appears to have zero lines, 1 line, or more than 1 line of symmetry. Write zero, 1, or more than 1.

1.



2.





Problem Solving • Shape Patterns

Use the strategy act it out to solve pattern problems.

What might be the next three figures in the pattern below?



Read the Problem

What do I need to find?

I need to find the next three figures in the pattern.

What information do I need to use?

I need to look for a group of figures that repeat.

How will I use the information?

I will use pattern blocks to model the pattern and act out the problem.

Solve the Problem

Look for a group of figures that repeat and circle that group.

The repeating group is triangle triangle square triangle square I used triangles and squares to model and continue the pattern by repeating the figures in the group.

These are the next three figures in the pattern:



0 00

000

0 00

1. Describe the pattern shown at right. Draw what might be the next figure in the pattern.

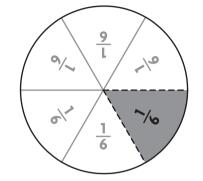


Angles and Fractional Parts of a Circle

Find how many $\frac{1}{6}$ turns make a complete circle.

Materials: fraction circles

Step 1 Place a $\frac{1}{6}$ piece so the tip of the fraction piece is on the center of the circle. Trace the fraction piece by drawing along the dashed lines in the circle.



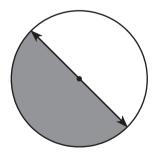
- **Step 2** Shade and label the angle formed by the $\frac{1}{6}$ piece.
- Step 3 Place the $\frac{1}{6}$ piece on the shaded angle. Turn it clockwise (in the direction that the hands on a clock move). Turn the fraction piece to line up directly beside the shaded section.
- **Step 4** Trace the fraction piece. Shade and label it. You have traced ____ sixths in all.
- Step 5 Repeat until you have shaded the entire circle.

There are <u>Six</u> angles that come together in the center of the circle.

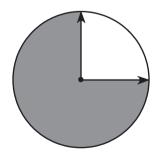
So, you need $\frac{\text{SiX}}{6}$ turns to make a circle.

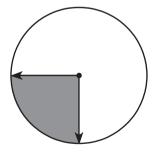
Tell what fraction of the circle the shaded angle represents.

1.



2.

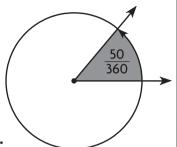




Degrees

Angles are measured in units called **degrees**. The symbol for degrees is °. If a circle is divided into 360 equal parts, then an angle that turns through 1 part of the 360 measures 1°.

An angle that turns through $\frac{50}{360}$ of a circle measures 50°.



Find the measure of an angle that turns through $\frac{1}{6}$ of a circle.

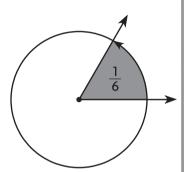
Step 1 Find a fraction that is equivalent to $\frac{1}{6}$ with 360 in the denominator. **Think:** $6 \times 60 = 360$.

$$\frac{1}{6} = \frac{1 \times 60}{6 \times 60} = \frac{60}{360}$$

Step 2 Look at the numerator of $\frac{60}{360}$.

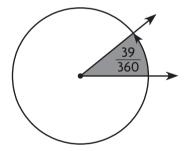
The numerator tells how many degrees are in $\frac{1}{6}$ of a circle.

So, an angle that turns through $\frac{1}{6}$ of a circle measures $\underline{60^{\circ}}$

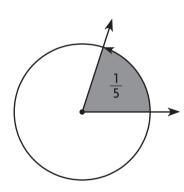


Tell the measure of the angle in degrees.

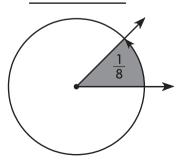
1.

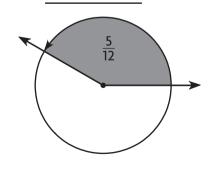


2.



3.





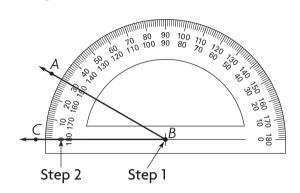
Measure and Draw Angles

A **protractor** is a tool for measuring the size of an angle.

Follow the steps below to measure $\angle ABC$.

Step 1 Place the center point of the protractor on vertex B of the angle.

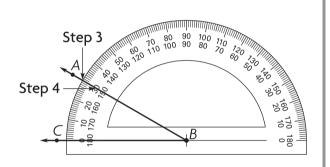
Step 2 Align the 0° mark on the protractor with ray BC. Note that the 0° mark is on the outer scale or top scale.



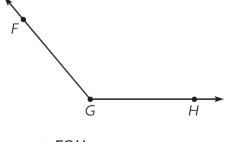
Step 3 Find where ray BA intersects the same scale.

Step 4 Read the angle measure on the scale.

The $m\angle ABC = 30^{\circ}$

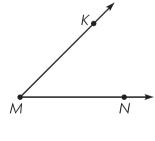


Use a protractor to find the angle measure.



m∠*FGH* _____

2.



m∠*KMN* _____

Use a protractor to draw the angle.

3. 110°

4. 55°

Join and Separate Angles

The measure of an angle equals the sum of the measures of its parts.

Use your protractor and the angles at the right.

Step 1 Measure $\angle ABC$ and $\angle CBD$. Record the measures.

$$m\angle ABC = \underline{35^{\circ}}; m\angle CBD = \underline{40^{\circ}}$$

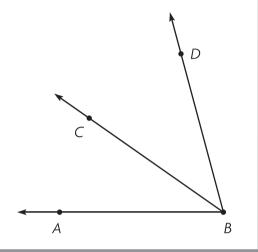
Step 2 Find the sum of the measures.

$$35^{\circ} + 40^{\circ} = 75^{\circ}$$

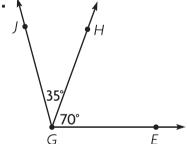
Step 3 Measure ∠*ABD*. Record the measure.

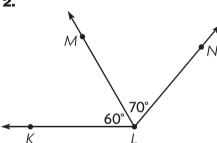
$$m\angle ABD = _{5}^{\circ}$$

So, $m\angle ABC + m\angle CBD = m\angle ABD$.

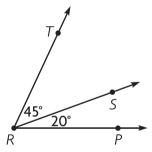


Add to find the measure of the angle. Write an equation to record your work.





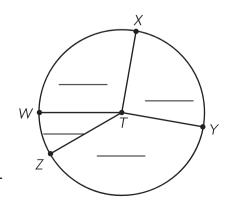
3.



$$m\angle EGJ = \underline{\qquad} \qquad m\angle KLN = \underline{\qquad}$$

Use a protractor and the art at the right.

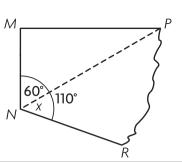
- 4. Find the measure of each angle. Label each angle with its measure.
- **5.** Write the sum of the angle measures as an equation.



Problem Solving • Unknown Angle Measures

Use the strategy draw a diagram.

Mrs. Allen is cutting a piece of wood for a set for the school play. She needs a piece of wood with a 60° angle. After the cut, what is the angle measure of the part left over?



Read the Problem

What do I need to find?

I need to find the angle

measure of the part left

over, or m∠PNR

What information do I need to use?

 $m \angle MNR = 110^{\circ}$

I can use the angle
measures I know:
m∠MNP = 60° and

How will I use the information?

I can draw a bar model to find the unknown angle measure, or m∠PNR

Solve the Problem

I can draw a bar model to represent the problem

Then I can write an equation to solve the problem

 $m \angle MNP + m \angle PNR = m \angle MNR$

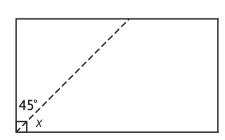
$$60^{\circ} + x = 110^{\circ}$$

$$x = 110^{\circ} - 60^{\circ}$$
, or 50°



So, $m\angle PNR = \underline{50^{\circ}}$

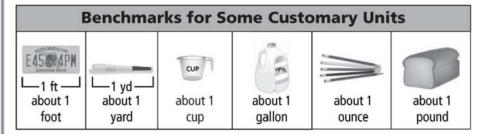
- **1.** Cal is cutting a rectangular board as shown. What is the angle measure of the part left over?
- 2. What equation did you use to solve?



Measurement Benchmarks

You can use benchmarks to estimate measurements.

The chart shows benchmarks for customary units of measurement.



Here are some more examples of estimating with customary units.

- The width of a professional football is about _______1 foot___.
- A large fish bowl holds about 1 gallon of water.
- A box of cereal weighs about 1 pound

The chart shows benchmarks for metric units of measurement.



Here are some more examples of estimating with metric units.

- The width of a large paper clip is about 1 centimeter
- A pitcher holds about __1 liter_ of juice.
- Three laps around a track is about <u>1 kilometer</u>

Use benchmarks to choose the customary unit you would use to measure each.

1. length of a school bus

2. weight of a computer

Use benchmarks to choose the metric unit you would use to measure each.

- the amount of liquid a bottle of detergent holds
- 4. distance between two cities

Customary Units of Length

A ruler is used to measure length. A ruler that is 1 foot long shows 12 inches in 1 foot. A ruler that is 3 feet long is called a yardstick. There are 3 feet in 1 yard.

How does the size of a foot compare to the size of an inch?

Step 1 A small paper clip is about 1 inch long. Below is a drawing of a chain of paper clips that is about 1 foot long. Number each paper clip, starting with 1.



Step 2 Complete this sentence.

In the chain of paper clips shown, there are _____ paper clips.

Step 3 Compare the size of 1 inch to the size of 1 foot.

There are $\frac{12}{}$ inches in $\frac{1}{}$ foot.

So, 1 foot is _____ times as long as 1 inch.

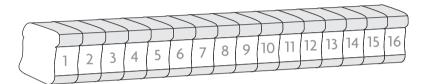
Customary Units of Weight

Ounces and **pounds** are customary units of weight. A **ton** is a unit of weight that is equal to 2,000 pounds.

A slice of bread weighs about 1 ounce. Some loaves of bread weigh about 1 pound.

How does the size of 1 ounce compare to the size of 1 pound?

Step 1 You know a slice of bread weighs about 1 ounce. Below is a drawing of a loaf of bread that weighs about 1 pound. Number each slice of bread, starting with 1.



Step 2 Complete this sentence.

In the loaf of bread shown above, there are ____16__ slices of bread.

Step 3 Compare the size of 1 ounce to the size of 1 pound.

There are $\underline{16}$ ounces in $\underline{1}$ pound.

So, 1 pound is <u>16</u> times as heavy as 1 ounce.

Complete.

Think: $2 \times 16 = 32$

Customary Units of Liquid Volume

Liquid volume is the measure of the space a liquid occupies. Some basic units for measuring liquid volume are gallons, half gallons, quarts, pints, cups, and fluid ounces. The table at the right shows the relationships among some units of liquid volume.

1 cup = 8 fluid ounces

1 pint = 2 cups

1 quart = 2 pints

1 half gallon = 2 quarts

1 gallon = 4 guarts

How does the size of a gallon compare to the size of a pint?

Step 1 Use the information in the table.

Draw a bar to represent 1 gallon.

1 gallon

Step 2 The table shows that 1 gallon is equal to 4 quarts. Draw a bar to show 4 quarts.

1 quart

1 quart

1 quart

1 quart

Step 3 The table shows that 1 quart is equal to 2 pints. Draw a bar to show 2 pints for each of the 4 quarts.

1 pint 1 pint 1 pint 1 pint 1 pint 1 pint 1	t 1 pint	1 pint 1 pin	1 pint					
---	----------	--------------	--------	--------	--------	--------	--------	--

Step 4 Compare the size of 1 gallon to the size of 1 pint.

There are 8 pints in 1 gallon.

So, 1 gallon is ___8 __ times as much as 1 pint.

Complete. Draw a model to help.

Line Plots

Howard gave a piece of paper with several survey questions to his friends. Then he made a list to show how long it took for his friends to answer the survey. Howard wants to know how many surveys took longer than $\frac{2}{10}$ hour.

Make a line plot to show the data.

Step 1 Order the data from least to greatest.

$$\frac{1}{12}$$
, $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{3}{12}$, $\frac{5}{12}$, $\frac{6}{12}$

Step 2 Make a tally table of the data.

Step 3 Label the fractions of an hour on the number line from least to greatest. Notice that $\frac{4}{12}$ is included even though it is not in the data.

Step 4 Plot an *X* above the number line for each piece of data. Write a title for the line plot.

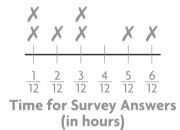
Step 5 Count the number of Xs that represent data points greater than $\frac{2}{12}$ hour.

There are $\frac{4}{12}$ data points greater than $\frac{2}{12}$ hour.

So, $\underline{\underline{}}$ surveys took more than $\frac{2}{12}$ hour.

Time for Survey Answers (in hours)

Surve	У
Time (in hours)	Tally
1/12	
<u>2</u> 12	
<u>3</u> 12	
<u>5</u> 12	
<u>6</u> 12	



Use the line plot above for 1 and 2.

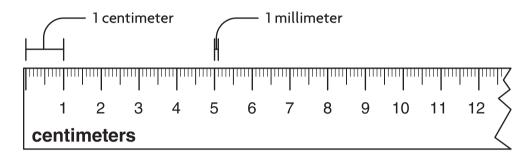
- **1.** How many of the surveys that Howard gave to his friends were answered?
- **2.** What is the difference in hours between the longest time and the shortest time that it took Howard's friends to answer the survey?

Metric Units of Length

Meters (m), **decimeters** (dm), centimeters (cm), and **millimeters** (mm) are all metric units of length. You can use a ruler and a meterstick to find out how these units are related.

Materials: ruler, meterstick

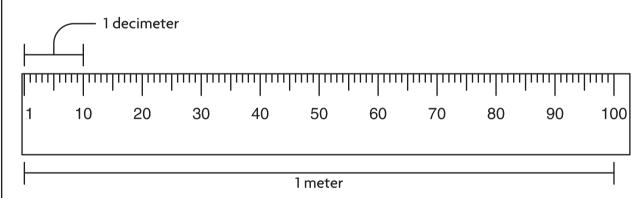
Step 1 Look at a metric ruler. Most look like the one below.



The short marks between each centimeter mark show millimeters.

1 centimeter has the same length as a group of 10 millimeters.

Step 2 Look at a meterstick. Most look like the one below.



1 decimeter has the same length as a group of 10 centimeters.

Step 3 Use the ruler and the meterstick to compare metric units of length.

1 centimeter =
$$\underline{10}$$
 millimeters 1 decimeter = $\underline{10}$ centimeters

1 meter =
$$\underline{10}$$
 decimeters

1 meter =
$$\frac{100}{100}$$
 centimeters

Metric Units of Mass and Liquid Volume

Mass is the amount of matter in an object. Metric units of mass include grams (g) and kilograms (kg). 1 kilogram represents the same mass as 1,000 grams.

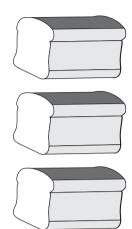
One large loaf of bread has a mass of about 1 kilogram. Jacob has 3 large loaves of bread. About how many grams is the mass of the loaves?

3 kilograms =
$$3 \times 1,000$$
 grams = $3,000$ grams



A large bowl holds about 2 liters of juice. Carmen needs to know the liquid volume in milliliters.

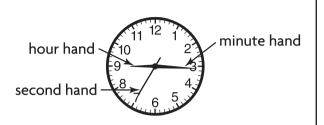
2 liters =
$$2 \times 1,000$$
 milliliters
= $2,000$ milliliters



Units of Time

Some analog clocks have an hour hand, a minute hand, and a **second** hand.

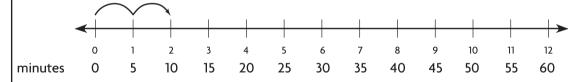
There are 60 seconds in a minute. The second hand makes 1 full turn every minute. There are 60 minutes in an hour. The minute hand makes 1 full turn every hour. The hour hand makes 1 full turn every 12 hours.



You can think of the clock as unrolling to become a number line.



The hour hand moves from one number to the next in 1 hour.



The minute hand moves from one number to the next in 5 minutes.

Use the table at the right to change between units of time.

1 hour = 60 minutes, or 60 \times 60 seconds, or 3,600 seconds.

So, 1 hour is 3,600 times as long as 1 second.

1 day = 24 hours, so 3 days = 3×24 hours, or _____ hours.

Units of Time

1 minute = 60 seconds

1 hour = 60 minutes

1 day = 24 hours

1 week = 7 days

1 year = 12 months

1 year = 52 weeks

Problem Solving • Elapsed Time

Opal finished her art project at 2:25 P.M. She spent 50 minutes working on her project. What time did she start working on her project?

	Read the Probler	n
What do I need to find?	What information do I need to use?	How will I use the information?
I need to find	End time: 2:25 P.M.	I can draw a diagram of a clock.
Opal's start time.	Elapsed time: minutes	I can then count back 5 minutes at a time until I reach 50 minutes.

Solve the Problem

I start by showing 2:25 $\ensuremath{\text{P.M.}}$ on the clock.

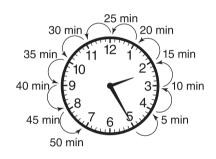
Then I count back 50 minutes by 5s.

Think: As I count back, I go past the 12.

The hour must be 1 hour less than the ending time.

The hour will be 1 o'clock

So, Opal started on her project at ____1:35 P.M._



Draw hands on the clock to help you solve the problem.

1. Bill wants to be at school at 8:05 A.M. It takes him 20 minutes to walk to school. At what time should Bill leave his house?

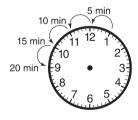
Bill should leave his house at ______.

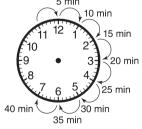
2. Mr. Gleason's math class lasts 40 minutes. Math class starts at 9:55 A.M. At what time does math class end?

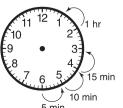
Math class ends at ______

3. Hannah rode her bike for 1 hour and 15 minutes until she got a flat tire at 2:30 P.M. What time did Hannah start riding her bike?

Hannah started riding her bike at _____







Mixed Measures

Gabrielle's puppy weighs 2 pounds 7 ounces. What is the weight of the puppy in ounces?

Step 1 Think of 2 pounds 7 ounces as 2 pounds + 7 ounces.

Step 2 Change the pounds to ounces.

Think: 1 pound =
$$\frac{16}{}$$
 ounces

So, 2 pounds = 2×16 ounces, or <u>32</u> ounces.

Step 3 Add like units to find the answer.

32 ounces + 7 ounces

So. Gabrielle's puppy weighs 39 ounces.

39 ounces

Gabrielle played with her puppy for 2 hours 10 minutes yesterday and 1 hour 25 minutes today. How much longer did she play with the puppy yesterday than today?

Step 1 Subtract the mixed measures. Write the subtraction with like units lined up.

2 hr 10 min

Think: 25 minutes is greater than 10 minutes.

- 1 hr 25 min

Step 2 Rename 2 hours 10 minutes to subtract.

So, 2 hr 10 min = 1 hr + 60 min + 10 min, or $\frac{1}{2}$ hr $\frac{70}{2}$ min. $\frac{2}{2}$ hr $\frac{10}{2}$ min.

70 - 1 hr 25 min

0 hr 45 min

Step 3 Subtract like units.

$$1 \text{ hr} - 1 \text{ hr} = 0 \text{ hr}$$
; $70 \text{ min} - 25 \text{ min} = \frac{45 \text{ min}}{1000 \text{ min}}$

So, she played with the puppy 45 minutes longer yesterday than today.

Complete.

Add or subtract.

Algebra • Patterns in Measurement Units

Use the relationship between the number pairs to label the columns in the table.

?	?
1	8
2	16
3	24
4	32

Step 1 List the number pairs. 1 and 8; 2 and 16; 3 and 24; 4 and 32

Step 2 Describe the relationship between the numbers in each pair.

The second number is 8 times as great as the first number.

Step 3 Look for a relationship involving 1 and 8 in the table below.

Length	Weight	Liquid Volume	Time
1 foot = 12 inches	1 pound = 16 ounces	1 cup = 8 fluid ounces	1 minute = 60 seconds
1 yard = 3 feet	1 ton = 2,000 pounds	1 pint = 2 cups	1 hour = 60 minutes
1 yard = 36 inches		1 quart = 2 pints	1 day = 24 hours
		1 gallon = 4 quarts	1 week = 7 days
			1 year = 12 months
			1 year = 52 weeks

Each table shows a pattern for two customary units. Label the columns of the table.

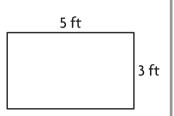
1.

	1	12		
	2	24		
	3	36		
	4	48		

٠.	`				
	1	2,000			
	2	4,000			
	3	6,000			
	4	8,000			

Perimeter

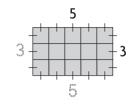
Perimeter is the distance around a shape. You can use grid paper to count the number of units around the outside of a rectangle to find its perimeter.



How many feet of ribbon are needed to go around the bulletin board?

Step 1 On grid paper, draw a rectangle that has a length of 5 units and a width of 3 units.

Step 2 Find the length of each side of the rectangle. Mark each unit of length as you count.



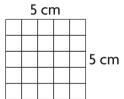
Step 3 Add the side lengths. 5 + 3 + 5 + 3 = 16

The perimeter is _____16__ feet.

So, 16 feet of ribbon are needed to go around the bulletin board.

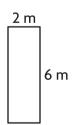
1. What is the perimeter of this square?

+___+ centimeters

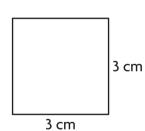


Find the perimeter of the rectangle or square.

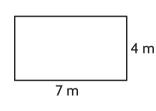
2.



3.



4.



meters

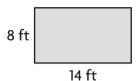
centimeters

meters

Area

Area is the number of **square units** needed to cover a flat surface.

Find the area of the rectangle at the right.



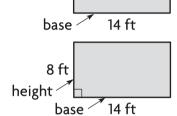
You can use the formula **Area** = **base** \times **height**.

Step 1 Identify one side as the base.

The base is ____14__ feet.

Step 2 Identify a perpendicular side as the height.

The height is ___8__ feet.



8 ft

Step 3 Use the formula to find the area.

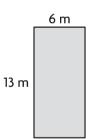
Area = base
$$\times$$
 height = 14 \times 8

= 112

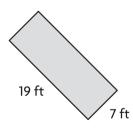
So, the area of the rectangle is 112 square feet.

Find the area of the rectangle or square.

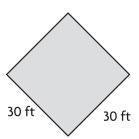
1.



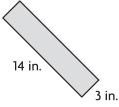
2.



3.

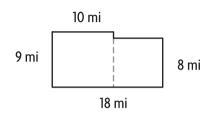


4.



Area of Combined Rectangles

Find the area of the combined rectangles.



Step 1 First, find the area of each section of the shape.

LEFT
$$A = b \times h$$

$$= 10 \times 9$$

$$= 90$$

RIGHT
$$A = b \times h$$

$$= 8 \times 8$$
$$= 64$$

Think:
$$18 - 10 = 8$$

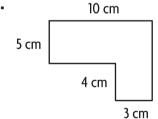
Step 2 Add the two areas.

$$90 + 64 = 154$$

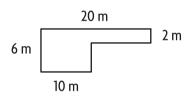
So, the total area is ______ square miles.

Find the area of the combined rectangles.

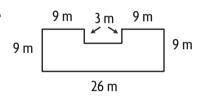
1.



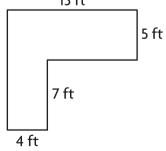
2.



3.



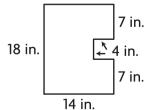
13 ft



5.



6.



Find Unknown Measures

Fred has 30 yards of fencing to enclose a rectangular vegetable garden. He wants it to be 6 yards wide. How long will his vegetable garden be?



Step 1 Decide whether this problem involves area or perimeter.

Step 2 Use a formula for perimeter. The width is 6. The perimeter is 30. The length is unknown.

Step 3 Find the value of *l*.

Think: The fencing goes around the outside of the garden. This is a measure of perimeter.

$$P = (2 \times I) + (2 \times w)$$

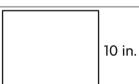
$$30 = (2 \times I) + (2 \times 6)$$

$$30 = 2 \times I + 12$$

 $18 = 2 \times I$, so the value of I is 9.

The length of Fred's garden will be 9 yards.

Carol has 120 square inches of wood. The piece of wood is rectangular and has a height of 10 inches. How long is the base?



Step 1 Decide whether this problem involves area or perimeter.

Step 2 Use a formula for area. The height is 10. The area is 120. The length is unknown.

Step 3 Find the value of b.

The base of Carol's piece of wood is 12 inches.

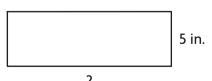
Think: Square inches is a measure of area.

$$A = b \times h$$
$$120 = b \times 10$$

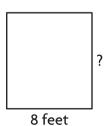
Since $120 = 12 \times 10$, the value of b is 12.

Find the unknown measure.

1.



2.



Perimeter = 40 inches

Area = 72 square feet

width = _____

height = _____

Problem Solving • Find the Area

Use the strategy solve a simpler problem.

Marilyn is going to paint a wall in her bedroom. The wall is 15 feet long and 8 feet tall. The window takes up an area 6 feet long and 4 feet high. How many square feet of the wall will Marilyn have to paint?

Read the Problem	Solve the Problem
What do I need to find?	First, find the area of the wall.
I need to find how many square feet of the wall Marilyn will paint.	$A = b \times h$ $= 15 \times 8$ $= 120 \text{ square feet}$
What information do I need to use?	Next, find the area of the window.
The paint will cover the wall. The paint will not cover the window. The base of the wall is 15 feet and the height is 8 feet. The base of the window is 6 feet and the height is 4 feet.	$A = b \times h$ $= \underline{6} \times \underline{4}$ $= \underline{24}$ square feet Last, subtract the area of the window from the area of the wall.
How will I use the information? I can solve simpler problems. Find the area of the wall. Then, find the area of the window. Last, subtract the area of the window from the area of the wall.	120 - 24 - 96 square feet So, Marilyn will paint 96 square feet of her bedroom wall.

- 1. Ned wants to wallpaper the wall of his bedroom that has the door. The wall is 14 feet wide and 9 feet high. The door is 3 feet wide and 7 feet high. How many square feet of wallpaper will Ned need for the wall?
- 2. Nicole has a rectangular canvas that is 12 inches long and 10 inches wide. She paints a blue square in the center of the canvas. The square is 3 inches on each side. How much of the canvas is NOT painted blue?

Add Dollars and Cents

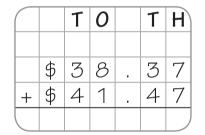
To add money amounts, line up the decimal points and then add as with whole numbers.

Find the sum.

\$38.37 + \$41.47

Step 1

Write the problem on grid paper. Align the digits by place value. Think of pennies as hundredths and dimes as tenths.



So, \$38.37 + \$41.47 = \$79.84.

Step 2

Add the hundredths. Regroup 14 hundredths as 1 tenth 4 hundredths. Write 1 in the tenths column.

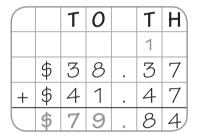
Then add the tenths.

	T	0	T	H
			1	
	3	8	3	7
+	\$ 4	1	4	7
			8	4

Step 3

Add the ones and then add the tens. Regroup if necessary.

Write the decimal point and dollar sign.



Find the sum.

Subtract Dollars and Cents

You can count up to find a difference.

Find the difference.

Step 1 Start with \$12.50, the amount being subtracted. Count up until you reach \$48.32. Record each amount that you use to count up.

+\$0	O.50 +	\$7 + \$	528 +\$6	0.32
\$12.50	\$13	\$20	\$48	\$48.32

Step 2 Add the distances between counts to find the difference.

$$$0.50 + $7.00 + $28.00 + $0.32 = $35.82$$

So,
$$$48.32 - $12.50 = $35.82$$
.

Find the difference.

Algebra • Order of Operations

The **order of operations** is a set of rules that gives the order in which calculations are done in an expression.

Use the order of operations to find the value of the expression. Show each step.

- 1. First, perform operations inside the parentheses.
- 2. Then, multiply and divide from left to right.
- **3.** Last, add and subtract from left to right.

$$8 + (10 \div 5) - 4$$

Step 1

First divide.

Think: $10 \div 5 = 2$

$$8 + (10 \div 5) - 4$$

 $8 + 2 - 4$

So,
$$8 + (10 \div 5) - 4 = 6$$
.

Step 2

Then add and subtract from left to right.

Think: 8 + 2 = 10

$$8 + 2 - 4$$
 $10 - 4$

Step 3

Subtract.

$$10 - 4 = 6$$

Write correct if the operations are listed in the correct order. If not correct, write the correct order of operations.

1.
$$(9 \div 3) \times 4$$

multiply, divide **2.**
$$15 - (8 \div 2)$$
 subtract, divide

3.
$$(36 + 10) \times 3$$
 multiply, add

4.
$$(16 - 4) \div 2 + 5$$
 subtract, divide, add

Follow the order of operations to find the value of the expression. Show each step.

5.
$$(6 \times 7) + 3$$

6.
$$(8 + 12) \div 4$$

7.
$$(20-5) \times 3 + 4$$

5.
$$(6 \times 7) + 3$$
 6. $(8 + 12) \div 4$ **7.** $(20 - 5) \times 3 + 4$ **8.** $18 + 6 + (16 \div 4)$

Divide by Multiples of Ten

You can use basic facts and patterns to divide by multiples of ten.

Divide. Use a pattern to help.

 $6.000 \div 30$

Step 1

Look for a basic fact.

 $6.000 \div 30$

Think: 6 ÷ 3

The basic fact is $6 \div 3 = 2$.

Step 2

Use the basic fact to find a division sentence with the same divisor as the original problem.

6,000 ÷ **30 ←** divisor

Think:
$$6 \div 3 = 2$$
, so $60 \div 30 = 2$.

Step 3

Now look for a pattern.

Think: If the number of zeros in the dividend increases. the number of zeros in the quotient increases by the same number.

dividend→
$$60 \div 30 = 2$$
 quotient
 $600 \div 30 = 20$
 $6,000 \div 30 = 200$

Divide. Use a pattern to help.

7.
$$6,000 \div 10 =$$
 8. $4,900 \div 70 =$ **9.** $5,400 \div 60 =$ **9.**

Model Division with 2-Digit Divisors

You can use models to divide a whole number by a 2-digit divisor.

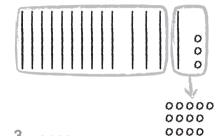
Use base-ten blocks to find 143 ÷ 13.

Step 1 Use base-ten blocks to model the dividend, <u>143</u>. Show 143 as 1 hundred 4 tens 3 ones.

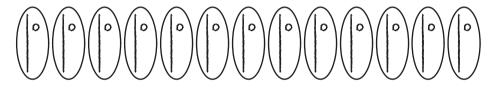


Remember: Each large square represents 100, each line represents 10, and each small circle represents 1.

Step 2 The divisor is <u>13</u>. Divide the blocks equally between 13 groups. Since you cannot share the one-hundred square equally between the 13 groups, first break it into 10 tens. Then you will have <u>14</u> tens, altogether. Share the tens equally among 13 groups.



Step 3 After completing Step 2, you will have __1_ ten and _3_ ones left over. Since you cannot share the 10 equally between the 13 groups, break it into 10 ones. Then you will have __13_ ones, altogether. Share the 13 ones equally among the 13 groups.



Each group contains $\frac{1}{1}$ ten and $\frac{1}{1}$ one, or 11. So 143 \div 13 = $\frac{11}{1}$.

Use base-ten blocks to divide.

Place Value Through Millions

You can use a place-value chart to help you read and write numbers through millions.

You can group the digits in a whole number into sections called periods. Each period has 3 digits.

Each digit in a whole number has both a place and a value. In the place value chart below, the digit 3 is in the hundred thousands place. So its value is $3 \times 100,000$, or 300,000.

Periods								
Millions Thousands Ones								
Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
2	8	7	3	1	4	6	5	9

Use the place-value chart to read and write the number in standard form, word form, and expanded form.

Standard Form: 287,314,659

Word Form: two hundred eighty-seven million, three hundred fourteen thousand, six hundred fifty-nine

Expanded Form: 200.000.000 + 80.000.000 + 7.000.000 +

300.000 + 10.000 + 4.000 + 600 + 50 + 9

Read and write the number in two other forms.

hundred twenty-nine

Decimals and Place Value

You can write decimals, like whole numbers, in standard form, word form, and expanded form.

In a place-value chart, whole numbers are to the left of the decimal point. Decimal amounts are to the right of the decimal point. The value of each place is one-tenth, or $\frac{1}{10}$, of the place to its left.

When you write a decimal in word form, write the decimal point as "and."

Write the decimal 12.34 in word form and expanded form.

Start by writing 12.34 in a place-value chart. First, align the decimal point with the decimal in the chart. Then place the digits.

Hundreds	Tens	Ones	Tenths	Hundredths
	1	2	3	4
	×10	_2_×1	3 × 1/10	×
	10	2	3 10	4 100

Word form: 12.34← Two decimals indicate hundredths.

Twelve and thirty-four hundredths

Expanded Form: Use the last row of the chart to help you write the decimal in expanded form.

$$12.34 = 10 + __2 + _{0.3} + 0.04$$

Read and write the decimal in two other forms.

1. eight and seven tenths

2.
$$10 + 3 + 0.9 + 0.05$$

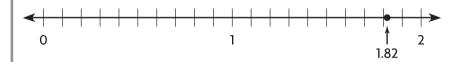
Round Decimals

You can use a number line to round whole numbers. You can also use a number line to round decimals.

Round 1.82 to the nearest whole number.

Step 1 Find the whole numbers it is between. 1 < 1.82 < 2

Step 2 Plot the number on a number line. See which whole number it is closest to.



1.82 is closer to ____ than ____.

So, 1.82 rounded to the nearest whole number is 2.

Note: If you are rounding a number with 5 in the tenths place, round the number to the greater whole number.

1.5 rounded to the nearest whole number is 2. However, 1.49 rounds to 1.

Round to the nearest dollar or to the nearest whole number.

- **1.** \$1.23
- **2.** 3.7

- **3.** 7.12
- **4.** \$5.50

- **5.** \$2.89
- **6.** 9.2

- **7.** \$4.49
- **8.** 6.51

9. 8.5

- **10.** \$5.01
- **11.** \$1.89
- **12.** \$8.21

Place Value to Compare Decimals

You can use a place-value chart to help you compare decimals.

Use a place-value chart to compare the decimals. Write <, >, or =.

4.28 4.23

- **Step 1** Write both decimals in a place-value chart. Line up each place and the decimal.
- **Step 2** Compare the numbers in each place, starting with the numbers in the ones place and working your way right.

Ones	Tenths	Hundredths
4	2	8
4	2	3

Step 3 Since 8 is greater than 3, 4.28 is greater than 4.23.

So. 4.28 > 4.23.

1. Use the place-value chart below to compare the decimals. Write <, >, or =.

Ones	•	Tenths	Hundredths
8		9	2
8		9	7

Compare the decimals. Write <, >, or =.

Decompose Multiples of 10, 100, 1,000

You can decompose a multiple of 10, 100, or 1,000 by finding its factors.

- To decompose a multiple of 10: rewrite it as the product of 10 and another number.
- To decompose a multiple of 100: rewrite it as the product of 100 and another number.
- To decompose a multiple of 1,000: rewrite it as the product of 1,000 and another number.

Decompose 3,200.

One Way Use mental math and a pattern.

$$3.200 = 3.200 \times 1$$

$$3,200 = 320 \times 10$$

$$3,200 = 32 \times 100$$

So
$$3.200 = 32 \times 100$$
.

Another Way Use place value.

$$3,200 = 32 \text{ hundreds} = 32 \times 100$$

So
$$3,200 = 32 \times 100$$
.

1. Complete the exercise below to decompose 3,600.

2. Complete the exercise below to decompose 870.

$$870 = tens = \times$$

Decompose each number.

Number Patterns

You already know how to use a rule and the first term to write a sequence. Now you will use multiplication to describe a pattern.

Stephen is saving his money to buy a car. The table shows how much money he has saved at the end of each month. If the pattern continues, how much money will Stephen have saved after months 5 and 6?

Number of Months	1	2	3	4
Total Amount Saved (\$)	15	30	60	120

Step 1 Describe the sequence.

Think: How do I get from one term to the next?

Try multiplying by 2, since $15 \times 2 = 30$.

Step 2 Write a rule that describes how much money Stephen has saved at the end of each month.

Rule: Multiply by 2.

Step 3 Use the rule to find the next two terms in the sequence.

So, at the end of month 5, Stephen will have saved \$240. At the end of month 6, have will have saved \$480.

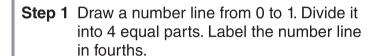
Describe the pattern. Then find the next two numbers in the pattern.

1. 2, 10, 50, ______, _____ **2.** 2, 6, 18, ______, ____

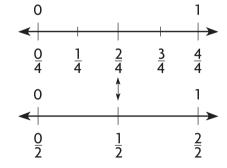
Add Related Fractions

You can add fractions with different denominators using a number line. First find an equivalent fraction so that both fractions have the same denominator.

Add $\frac{1}{4} + \frac{1}{2}$. Use a number line to help.



Step 2 Draw another number line directly below the first number line. Line up the 0s and 1s. Divide the second number line into 2 equal parts. Label it.



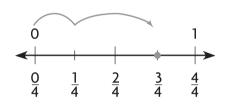
Step 3 Find how many fourths are equal to $\frac{1}{2}$.

$$\frac{1}{2} = \frac{2}{4}$$

Step 4 Add. Use the equivalent fraction for $\frac{1}{2}$.

$$\frac{1}{4} + \frac{1}{2} = \frac{1}{4} + \frac{2}{4}$$
$$= \frac{3}{4}$$

So,
$$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$
.



Add. Use a number line to help.

1.
$$\frac{3}{4} + \frac{1}{8} =$$

2.
$$\frac{1}{6} + \frac{2}{3} =$$

3.
$$\frac{2}{5} + \frac{1}{10} =$$

4.
$$\frac{4}{8} + \frac{1}{2} =$$

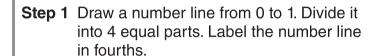
5.
$$\frac{1}{2} + \frac{1}{6} =$$

6.
$$\frac{4}{12} + \frac{1}{4} =$$

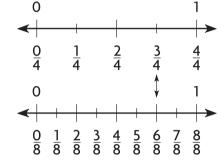
Subtract Related Fractions

You can subtract fractions with different denominators using a number line. First find an equivalent fraction so that both fractions have the same denominator.

Subtract $\frac{3}{4} - \frac{1}{8}$. Use a number line to help.



Step 2 Draw another number line directly below the first number line. Line up the 0s and 1s. Divide the second number line into 8 equal parts. Label it.



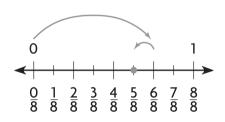
Step 3 Find how many eighths are equal to $\frac{3}{4}$.

$$\frac{3}{4} = \frac{6}{8}$$

Step 4 Add. Use the equivalent fraction for $\frac{3}{4}$.

$$\frac{3}{4} - \frac{1}{8} = \frac{6}{8} - \frac{1}{8}$$
$$= \frac{5}{8}$$

So,
$$\frac{3}{4} - \frac{1}{8} = \frac{5}{8}$$
.



Subtract. Use a number line to help.

1.
$$\frac{5}{6} - \frac{1}{3} =$$

2.
$$\frac{1}{2} - \frac{1}{4} =$$

3.
$$\frac{5}{8} - \frac{1}{2} =$$

4.
$$\frac{6}{10} - \frac{2}{5} =$$

5.
$$\frac{7}{12} - \frac{1}{3} =$$

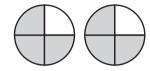
6.
$$\frac{6}{8} - \frac{3}{4} =$$

Compare Fraction Products

When a fraction less than one is multiplied by a whole number, is the product less than or greater than the fraction?

Is the product of $\frac{3}{4} \times 2$ less than or greater than $\frac{3}{4}$?

Step 1 Show two groups of $\frac{3}{4}$.



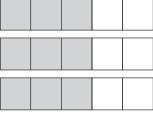
The model shows $\frac{6}{4}$ shaded. **Step 2** Compare. The product $\frac{6}{4}$ is **greater than** $\frac{3}{4}$.

So, the product of $\frac{3}{4} \times 2$ is greater than $\frac{3}{4}$.

When a whole number is multiplied by a fraction less than one, is the product less than or greater than the whole number?

Is the product of $3 \times \frac{3}{5}$ less than or greater than 3?

Step 1 Show three groups of $\frac{3}{5}$.



The model shows $\frac{9}{5}$ shaded. **Step 2** Compare. The product $\frac{9}{5}$ is <u>less than</u> 3.

So, the product of $3 \times \frac{3}{5}$ is less than 3.

Complete each statement with greater than or less than.

- **3.** $3 \times \frac{2}{5}$ will be ______3. **4.** $\frac{2}{3} \times 4$ will be ______ $\frac{2}{3}$.

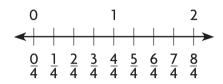
Repeated Subtraction with Fractions

You can use repeated subtraction to divide whole numbers.

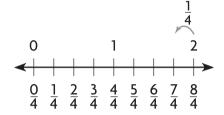
You can also use repeated subtraction to divide a whole number by a fraction.

Use repeated subtraction to find $2 \div \frac{1}{4}$.

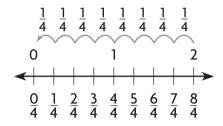
Step 1 Draw a number line from 0 to 2. Divide it into fourths.



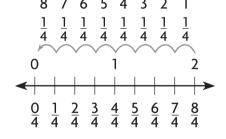
Step 2 Start at 2. Count back by $\frac{1}{4}$ to subtract.



Step 3 Keep subtracting $\frac{1}{4}$ until you reach 0 or get as close to it as possible.



Step 4 Count the number of times you counted back by $\frac{1}{4}$. You counted back 8 groups of $\frac{1}{4}$.



So, $2 \div \frac{1}{4} = 8$.

Use repeated subtraction to divide.

1.
$$3 \div \frac{1}{2}$$

2.
$$2 \div \frac{1}{5}$$

3.
$$1 \div \frac{1}{4}$$

4.
$$4 \div \frac{1}{3}$$

5.
$$2 \div \frac{1}{6}$$

6.
$$2 \div \frac{1}{8}$$

Fractions and Division

You can use division to make equal shares or to make equal-sized groups. You can use a fraction to show division.

Write the division problem as a fraction.

Think of a division sign as a fraction bar.

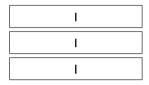
numerator ÷ denominator - numerator denominator

3 ÷ 4

You can use fraction strips to model the relationship between division and fractions.

Step 1

Begin with 3 wholes.



Step 2

Think of each whole as 4 fourth-size pieces.

<u> </u>	1/4	1/4	1/4
1/4	1/4	1/4	1/4
14	1/4	14	1/4

Step 3

Arrange the fourth-size pieces into 4 equal groups.

1/4	1/4	1/4	
<u> </u>	14	1/4	
<u> </u>	<u> </u>	<u> </u>	
<u> </u>	<u> </u>	<u> </u>	

There are 3 fourth-size pieces in each equal group.

So, $3 \div 4$ can be written as $\frac{3}{4}$.

Write the division problem as a fraction. Write each fraction greater than 1 as a whole number or mixed number.

Locate Points on a Grid

A map has horizontal and vertical lines that make a grid.

You can name a point on the grid using an **ordered pair** of numbers.

The first number tells \longrightarrow (1, 5) \longleftarrow The second number tells how how many units to move many units to move up from zero. right from zero.

Write the ordered pair for the location of the park.

Step 1 Start at zero. Move right. Count the number of units until you are directly below the park.

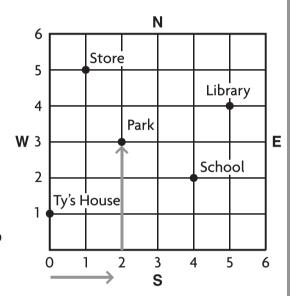
You move right 2 units.

Step 2 Move up. Count the number of units until you reach the park.

You move up 3 units.

Step 3 You move right 2 units and up 3 units, so the ordered pair is (2, 3).

So, the park is located at (2, 3) on the map.

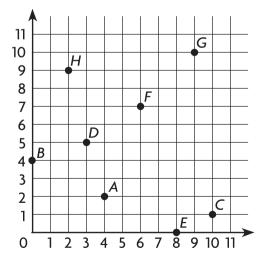


Use the grid. Write the ordered pair for each point.

- **1.** A
- **2.** B
- **3.** C
- **4.** D

Use the grid. Write the point for each ordered pair.

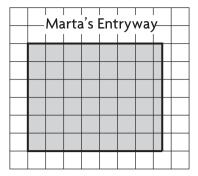
- **5.** (8, 0)
- **6.** (9, 10) **7.** (6, 7)
- **8.** (2, 9)



Area and Tiling

In the model, whole tiles are shaded, and some half tiles are shaded. You can combine the areas of half tiles and whole tiles to find the total area.

Find the area of the entryway. Write the area in square feet.



1 square = 4 square feet

Step 1 Count the number of whole tiles.
There are 42 whole tiles.

Step 2 Count the number of half tiles.
There are 6 half tiles.

Think: 2 half tiles = 1 whole tile 6 half tiles = 3 whole tiles

Step 3 Use the total number of whole tiles to find the area.

42 + 3 = 45 whole tiles

Think: 1 tile = 4 square feet

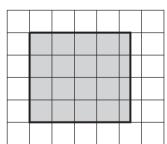
Multiply the number of whole tiles by 4 to find the area.

 $45 \times 4 = 180$

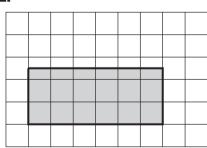
So, the area of Marta's entryway is 180 square feet.

Find the area of each shaded shape. Write the area in square units.

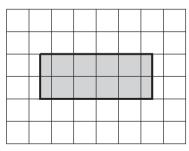
1.



2.



3.



1 square = 4 square feet

1 square = 9 square meters

1 square = 16 square miles

Multiply Three Factors

Step 1

Simplify the problem. Rewrite $2 \times (14 \times 6)$ as a product of two factors.

$$2 \times (14 \times 6) = 2 \times (\underline{6} \times 14)$$
 Commutative Property

$$= (2 \times \underline{6}) \times 14$$
 Associative Property

$$=$$
 $\underline{12}$ \times 14

So,
$$2 \times (14 \times 6) = 12 \times 14$$
.

Step 2

Multiply.

$$\begin{array}{ccc}
+ 120 & \longleftarrow & 10 \times 12 \\
\hline
 & 168 & \longleftarrow & Add.
\end{array}$$

So,
$$2 \times (14 \times 6) = 168$$
.

Remember

Commutative Property of Multiplication

You can multiply factors in any order and still get the same product.

Example: $2 \times 3 = 3 \times 2$

Associative Property of Multiplication

You can group factors in any order and still get the same product.

Example:

$$2\times(3\times4)=(2\times3)\times4$$

Find each product.

1.
$$3 \times (16 \times 4) =$$

2.
$$(4 \times 14) \times 6 =$$

1.
$$3 \times (16 \times 4) =$$
 2. $(4 \times 14) \times 6 =$ **3.** $5 \times (13 \times 5) =$

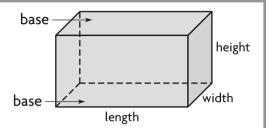
4.
$$(16 \times 7) \times 3 =$$

4.
$$(16 \times 7) \times 3 =$$
 5. $7 \times (18 \times 6) =$

6.
$$(12 \times 8) \times 6 =$$

Find Area of the Base

A **rectangular prism** is a solid figure that has three-dimensions: length, width, and height. A rectangular prism has two **bases**. The bases are the same size and shape and are opposite each other. The base shape of a rectangular prism is a rectangle or a square.



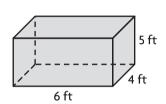
You can use the area formulas for a rectangle and a square to find the area of the base of a rectangular prism.

Find the area of the base of the rectangular prism.

Step 1 Identify the base shape.

The length is 6 feet.

The width is 4 feet.



The base shape is a rectangle.

Step 2 Find the area of the base shape.

 $A = I \times w$ Think: Use the area formula for a rectangle.

So, the area of the base is __24 square feet

Remember

Area of a rectangle:

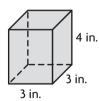
$$A = b \times h \text{ or } A = l \times w$$

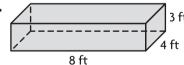
Area of a square:

$$A = s \times s$$

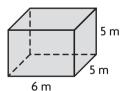
Find the area of the base of the rectangular prism.

1.

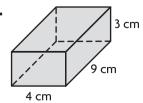


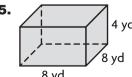


3.



4.





6.

